

MATLAB Projects

Problem 8.52

Digital speech and audio equalizer Design a seven-band audio equalizer using fourth-order bandpass filters with a sampling rate of 44.1 kHz. The center frequencies are listed in **Table 8.14**. In this project, use the designed equalizer to process a stereo audio ("No9seg.wav").

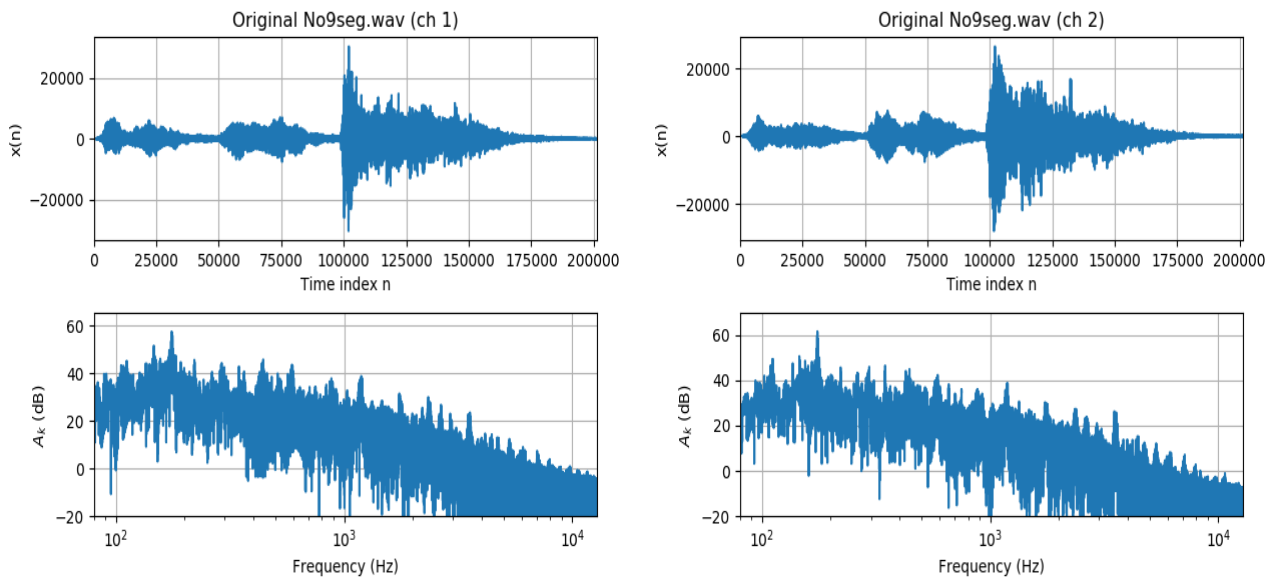
- Plot the magnitude response for each filter bank.
- Listen and evaluate the processed audio with the following gain settings:
 1. each filter bank gain=0 (no equalization)
 2. low-pass filtered
 3. band-pass filtered
 4. high-pass filtered

Table 8.14 Specification for Center Frequencies and Bandwidths

Center Frequency (Hz)	160	320	640	1280	2560	5120	10,240
Bandwidth (Hz)	80	160	320	640	1280	2560	5120

solution

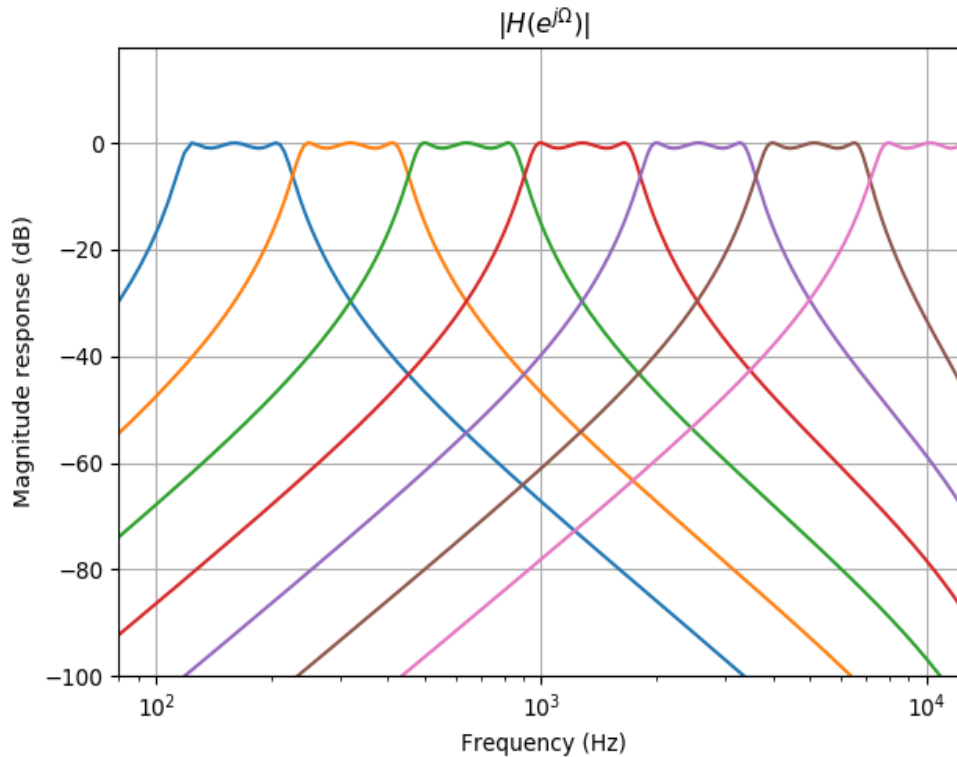
Original signal



Band-pass filter

Here we design the filter:

- Chebyshev I filter: order $n = 3$
- Pass band $A_p = 1$ dB
- Filter type: band-pass
- center frequency $f_c = [160, 320, 640, 1280, 2560, 5120, 10240]$
- Band-width $BW = [80, 160, 320, 640, 1280, 2560, 5120]$
- Sampling frequency $f_s = 44.1$ kHz



Design Chebyshev filter, $A_p = 1$ dB, $\varepsilon = \sqrt{10^{A_p/10} - 1} = 0.5088$

Now with order $n = 3$, $\varepsilon = 0.5088$, design the Chebyshev filter, n is odd

$$\begin{aligned}
 H(s') &= \frac{1}{\varepsilon 2^{n-1}} \frac{1}{(s + \text{sh}) \prod_{m=0}^{(n-1)/2-1} (s^2 + [2 \times \text{sh} \times s(m)]s + [\text{sh}^2 + 1 - s^2(m)])} \\
 &= 0.4913 \times \frac{1}{s + 0.4942} \times \frac{1}{s^2 + 0.4942s + 0.9942}
 \end{aligned}$$

Here, $\text{sh} \equiv \sinh\left(\frac{1}{n} \text{arsinh}\left(\frac{1}{\varepsilon}\right)\right)$, $s(m) \equiv \sin\left(\frac{\pi}{2}\left(\frac{1}{n}\right) + \pi\left(\frac{m}{n}\right)\right)$

Then substitute $s' = \frac{s(\omega_H - \omega_L)}{s^2 + \omega_H \omega_L}$, band-stop filter

Here with $\omega = (2f_s) \times \tan\left(\pi \frac{f}{f_s}\right) = (2f_s) \times \tan\left(\frac{2\pi f}{2f_s}\right)$,

we have $\omega_L = [754.0, 1508.11, 3017.1, 6041.28, 12139.51, 24747.84, 53725.46]$

and $\omega_H = [1340.5, 2681.49, 5367.0, 10766.21, 21794.31, 45827.05, 115719.83]$

$H(s)$

$$\begin{aligned} &= 0.4913 \times \frac{586.4957s}{1.0s^2 + 289.8289s + 1010735.0299} \\ &\times \frac{343977.2177s^2}{s^4 + 289.8289s^3 + 2363453.7874s^2 + 292940262.0727s + 1021585300762.115} \quad [1st] \\ &= 0.4913 \times \frac{1173.382s}{1.0s^2 + 579.8509s + 4043990.9005} \\ &\times \frac{1376825.3819s^2}{s^4 + 579.8509s^3 + 9456827.9109s^2 + 2344911788.4637s + 16353862403450.79} \quad [2nd] \\ &= 0.4913 \times \frac{2349.8925s}{1.0s^2 + 1161.2478s + 16192794.6791} \\ &\times \frac{5521994.934s^2}{s^4 + 1161.2478s^3 + 37875582.0497s^2 + 18803847462.5707s + 2.6221 \times 10^{14}} \quad [3rd] \\ &= 0.4913 \times \frac{4724.9293s}{s^2 + 2334.9212s + 65041670.5048} \\ &\times \frac{22324957.049s^2}{s^4 + 2334.9212s^3 + 152278915.7077s^2 + 151867173966.5871s + 4.2304 \times 10^{15}} \quad [4th] \\ &= 0.4913 \times \frac{9654.8014s}{s^2 + 4771.119s + 264572327.7615} \\ &\times \frac{93215190.0733s^2}{s^4 + 4771.119s^3 + 621819625.0523s^2 + 1262306072671.3513s + 6.9999 \times 10^{16}} \quad [5th] \\ &= 0.4913 \times \frac{21079.2088s}{s^2 + 10416.7253s + 1134120477.0311} \\ &\times \frac{444333041.5525s^2}{1.0s^4 + 10416.7253s^3 + 2709998902.0356s^2 + 11813821511707.322s + 1.2862 \times 10^{18}} \quad [6th] \\ &= 0.4913 \times \frac{61994.3763s}{s^2 + 30635.7984s + 6217101105.0179} \\ &\times \frac{3843302690.7742s^2}{1.0s^4 + 30635.7984s^3 + 16255231373.6225s^2 + 190465856278593.03s + 3.8652 \times 10^{19}} \quad [7th] \end{aligned}$$

Thus, $H(z)$

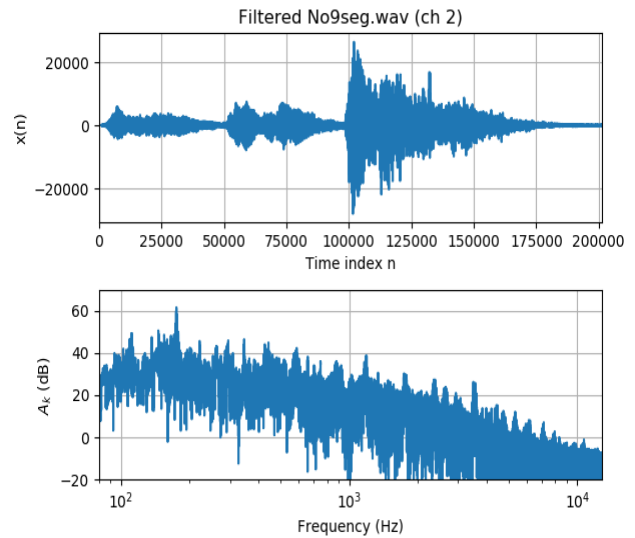
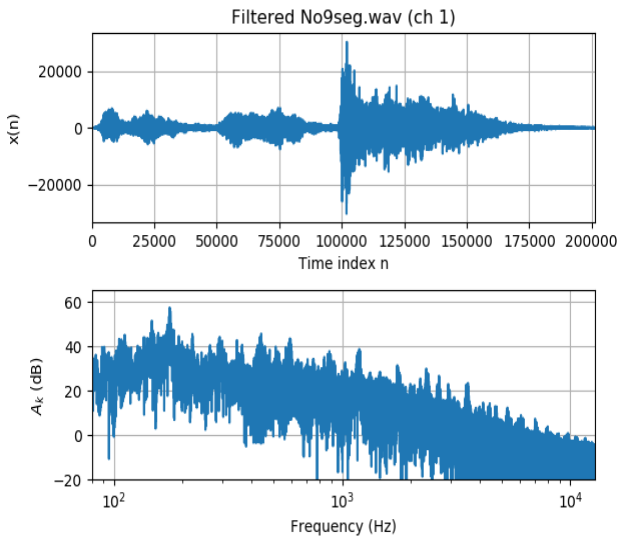
$$\begin{aligned} H(z) &= [(1.4345 \times 10^{-7} - 4.3035 \times 10^{-7} z^{-2} - 0.0 z^{-3} + 4.3035 \times 10^{-7} z^{-4} - 1.4345 \times 10^{-7} z^{-6}) \\ &/ (1.0 - 5.9852 z^{-1} + 14.9276 z^{-2} - 19.8588 z^{-3} + 14.8624 z^{-4} - 5.933 z^{-5} + 0.9869 z^{-6})] \quad [1st] \\ &= [(1.1398 \times 10^{-6} - 3.4194 \times 10^{-6} z^{-2} + 0.0 z^{-3} + 3.4194 \times 10^{-6} z^{-4} - 1.1398 \times 10^{-6} z^{-6}) \\ &/ (1.0 - 5.967 z^{-1} + 14.8421 z^{-2} - 19.6985 z^{-3} + 14.7127 z^{-4} - 5.8633 z^{-5} + 0.9741 z^{-6})] \quad [2nd] \\ &= [(8.9896 \times 10^{-6} - 2.6969 \times 10^{-5} z^{-2} - 0.0 z^{-3} + 2.6969 \times 10^{-5} z^{-4} - 8.9896 \times 10^{-6} z^{-6}) \\ &/ (1.0 - 5.9207 z^{-1} + 14.6327 z^{-2} - 19.3225 z^{-3} + 14.3786 z^{-4} - 5.7169 z^{-5} + 0.9488 z^{-6})] \quad [3rd] \\ &= [(6.9757 \times 10^{-5} - 0.0002093 z^{-2} + -0.0 z^{-3} + 0.0002093 z^{-4} - 6.9757 \times 10^{-5} z^{-6}) \\ &/ (1.0 - 5.7893 z^{-1} + 14.0683 z^{-2} - 18.3659 z^{-3} + 13.5846 z^{-4} - 5.3981 z^{-5} + 0.9004 z^{-6})] \quad [4th] \\ &= [(0.0005209 - 0.001563 z^{-2} + 0.0 z^{-3} + 0.001563 z^{-4} - 0.000521 z^{-6}) \\ &/ (1.0 - 5.3813 z^{-1} + 12.4453 z^{-2} - 15.7999 z^{-3} + 11.6086 z^{-4} - 4.6826 z^{-5} + 0.812 z^{-6})] \quad [5th] \\ &= [(0.0035 - 0.0 z^{-1} - 0.0106 z^{-2} + 0.0106 z^{-4} + 0.0 z^{-5} - 0.0035 z^{-6}) \\ &/ (1.0 - 4.0872 z^{-1} + 8.1194 z^{-2} - 9.5695 z^{-3} + 7.0896 z^{-4} - 3.1149 z^{-5} + 0.6669 z^{-6})] \quad [6th] \\ &= [(0.0183 - 0.0548 z^{-2} + 0.0548 z^{-4} - 0.0183 z^{-6}) \\ &/ (1.0 - 0.5511 z^{-1} + 2.0431 z^{-2} - 0.7891 z^{-3} + 1.6473 z^{-4} - 0.3362 z^{-5} + 0.4822 z^{-6})] \quad [7th] \end{aligned}$$

$$Y(z) \equiv X(z) + \sum_{k=1}^7 \text{Gain}_k \times H(z)_k X(z)$$

$$y(n) = x(n) + \sum_{k=1}^7 \text{Gain}_k \times [h_k(n) * x(n)]$$

no equalization

Gain = [0, 0, 0, 0, 0, 0, 0]

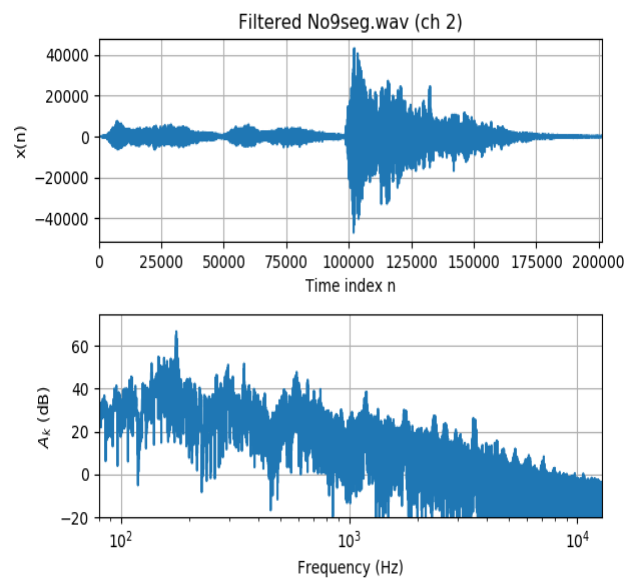
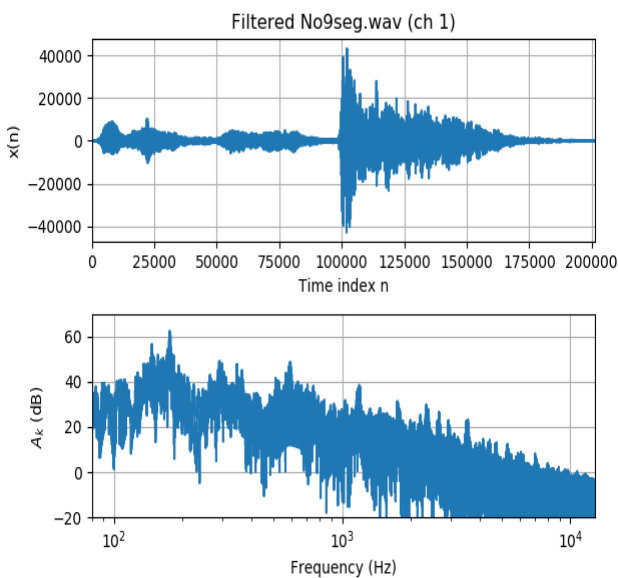


low-pass filtered

Gain = [1, 1, 1, 0, 0, 0, 0]

We can see there are 3 peaks [160, 320, 640] in the filtered spectrum, components in these band [80, 160, 320] are strengthened by the low-pass filter.

We can hear low frequency components more clearly.

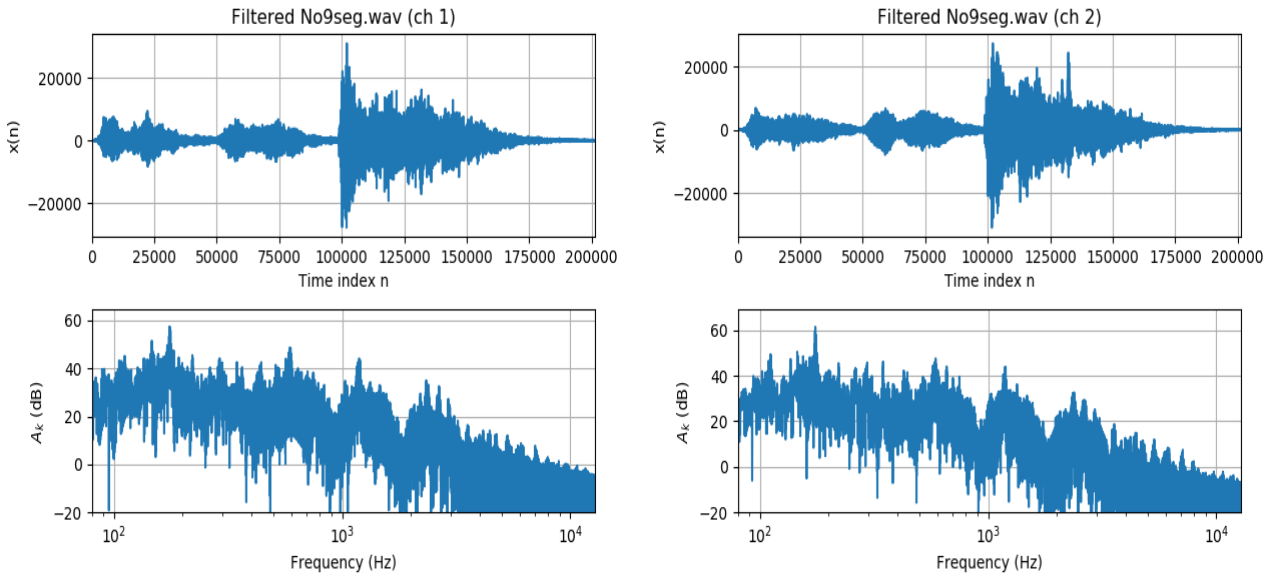


band-pass filtered

$$\text{Gain} = [0, 0, 1, 1, 1, 0, 0]$$

We can see there are 3 peaks[640, 1280, 2560] in the filtered spectrum, components in these band[320, 640, 1280] are strengthened by the band-pass filter.

We can hear the frequency components in specific frequency band more clearly.

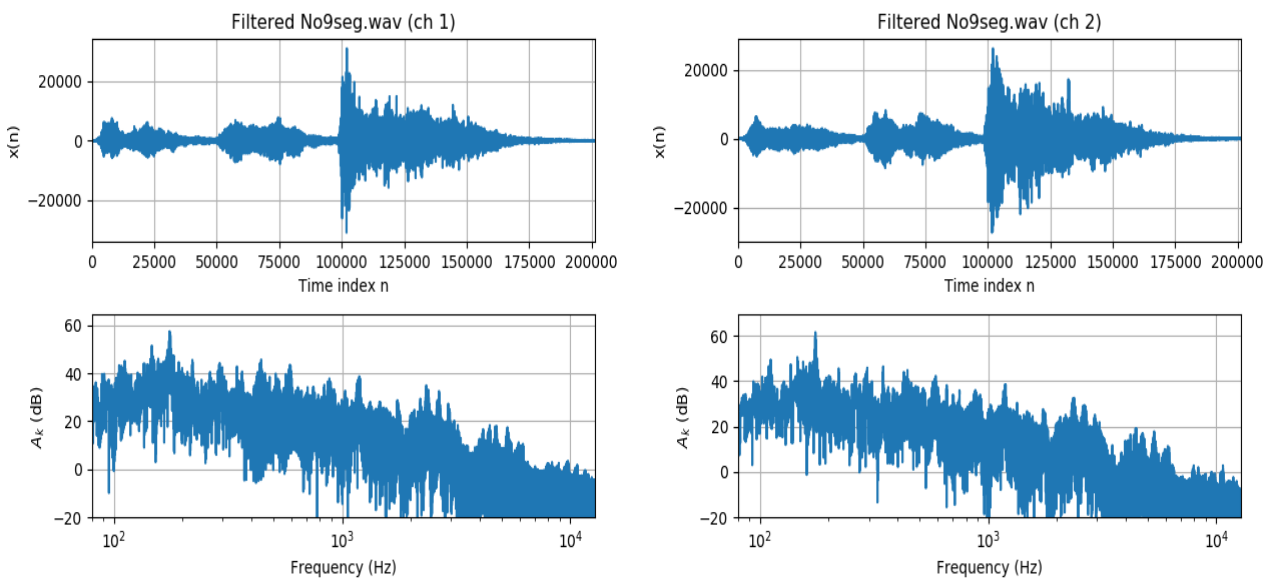


high-pass filtered

$$\text{Gain} = [0, 0, 0, 0, 1, 1, 1]$$

We can see there are 3 peaks[2560, 5120, 10240] in the filtered spectrum, components in these band[1280, 2560, 5120] are strengthened by the high-pass filter.

We can hear low frequency components more clearly.



Here is the main **Python** script with my IIR implementation library.

The **IIR library** implements:

- Calculation, substitution of Polynomial, Fraction of Polynomial $H(s), H(z)$
- BLT, unit low-pass filter $H(s')$ to low-pass, high-pass, band-pass, band-stop
- Magnitude $|H|$, Phase $\angle H$ of $H(s), H(z)$
- FFT to calculate A_k of $X(k), Y(k)$
- Butterworth, Chebyshev I filter: $H(s)$
- IIR filter: $y(n) = \sum_{k=0}^M b_k x(n-k) - \sum_{k=1}^N a_k y(n-k)$
- Pole-zero Placement parameters
- Plot of Impulse invariance

```
from scipy.io.wavfile import write, read # save sounds
from iir_filter.fft1d import plot_spectrum_dB
from iir_filter.frac import Frac, convert_s2z
from iir_filter.poly import Poly, Polyz
from iir_filter.util import convert_omega_z2s, filter_subs, calc_omega_pass
from iir_filter.calc_mag_angle import calc_mag_angle, plot_mag_freq_multiple
from iir_filter.protype import chebyshev_protype, calc_cheby_eps2
from iir_filter.iir_filter import iir_filter
from math import pi, sqrt, ceil
from functools import reduce

f_sample, list_input = read("./No9seg.wav") # sample rate, input
list_input_ch1, list_input_ch2 = list_input.T[0], list_input.T[1]
plot_spectrum_dB(list_input_ch1, f_sample, path_fig="./p8_52_input_ch1.png",
str_title="Original No9seg.wav (ch 1)")
plot_spectrum_dB(list_input_ch2, f_sample, path_fig="./p8_52_input_ch2.png",
str_title="Original No9seg.wav (ch 2)")

list_f_center = [160, 320, 640, 1280, 2560, 5120, 10240]
list_BW = [80, 160, 320, 640, 1280, 2560, 5120]
list_omega_pass_low = [2*pi*(f_center - 0.5 * BW) for f_center, BW in
list(zip(list_f_center, list_BW))]
list_omega_pass_high = [2*pi*(f_center + 0.5 * BW) for f_center, BW in
list(zip(list_f_center, list_BW))]
list2D_omega_pass_z = list(zip(list_omega_pass_low, list_omega_pass_high))
num_filter = len(list2D_omega_pass_z)
order = 3
A_p = 1
epsilon = sqrt( calc_cheby_eps2(A_p) )
print("epsilon = " + str(epsilon))
list_H_s = chebyshev_protype(order, epsilon)
print(list_H_s)
list_H_z = []
list2D_mag, list2D_omega = [], []
for ind in range(num_filter):
    list_omega_pass_z = list2D_omega_pass_z[ind]
    list_omega_pass_s = calc_omega_pass(list_omega_pass_z, f_sample,
str_filter_type="band_pass")
    print(list_omega_pass_s)
```

```

list_H_subs = [filter_subs(H_s, list_omega_pass_s, str_filter_type="band_pass") for
H_s in list_H_s]
print(list_H_subs)
H_subs = reduce(lambda x,y: x * y, list_H_subs)
print(H_subs)
H_z = convert_s2z(H_subs, f_sample)
print(H_z)
list_H_z.append(H_z)
list_mag, list_angle, list_omega = calc_mag_angle(H_z, num_point=4096)
list2D_mag.append(list_mag)
list2D_omega.append(list_omega)
plot_mag_freq_multiple(list2D_mag, list2D_omega, f_sample, path_fig="../p8_52_H_z.png")
# band_gain = [1] + [0, 0, 0, 0, 0, 0, 0, 0] # the first 1 represent original input gain: no
equalization
# band_gain = [1] + [1, 1, 1, 0, 0, 0, 0, 0] # low pass
# band_gain = [1] + [0, 0, 1, 1, 1, 0, 0, 0] # band pass
band_gain = [1] + [0, 0, 0, 0, 1, 1, 1, 1] # high pass
list2D_output_ch1 = [list_input_ch1]
list2D_output_ch2 = [list_input_ch2]
for H_z in list_H_z:
    list2D_output_ch1.append( iir_filter(list_input_ch1, H_z) )
    list2D_output_ch2.append( iir_filter(list_input_ch2, H_z) )
list2D_output_ch1 = list(map(list, zip(*list2D_output_ch1))) # transpose
list2D_output_ch2 = list(map(list, zip(*list2D_output_ch2)))
list_output_ch1, list_output_ch2 = [], []
for out_ch1, out_ch2 in list(zip(list2D_output_ch1, list2D_output_ch2)):
    list_output_ch1.append( int(sum([elem * gain for elem, gain in list(zip(out_ch1,
band_gain))])) )
    list_output_ch2.append( int(sum([elem * gain for elem, gain in list(zip(out_ch2,
band_gain))])) )
plot_spectrum_dB(list_output_ch1, f_sample, path_fig="../p8_52_output_high pass_ch1.png",
str_title="Filtered No9seg.wav (ch 1)")
plot_spectrum_dB(list_output_ch2, f_sample, path_fig="../p8_52_output_high pass_ch2.png",
str_title="Filtered No9seg.wav (ch 2)")
import numpy as np
list_output = np.asarray([list_output_ch1, list_output_ch2]).T
max_output = max(np.max(list_output), -np.min(list_output))
factor = (2**(16-1)/max_output)
list_output_scaled = np.floor(list_output * factor).astype(np.int16)
write("../No9seg_high pass.wav", f_sample, list_output_scaled)

```