# Homework 7

Course Title: Digital Signal Processing I (Spring 2020)

Course Number: ECE53800

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**Chapter 8**

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# Problems

## Problem 8.1

Given an analog filter with the transfer function

$$H(s) = \frac{1000}{s + 1000}$$

convert it to the digital filter transfer function and difference equation using the BLT if the DSP system has a sampling period of T=0.001 s.

### solution

$f_s = 1/T = 1000$ Hz

$$H(z) = \frac{1000}{s + 1000}\Big|_{s=2f_s\frac{1-z^{-1}}{1+z^{-1}}} = \frac{0.3333 + 0.3333z^{-1}}{1 - 0.3333z^{-1}}$$

The difference equation

$$\begin{gathered}[1 - 0.3333z^{-1}]Y(z) = [0.3333 + 0.3333z^{-1}]X(z) \\ y(n) - 0.3333y(n-1) = 0.3333x(n) + 0.3333x(n-1)\end{gathered}$$

Thus, $y(n) = 0.3333x(n) + 0.3333x(n-1) + 0.3333y(n-1)$

## Problem 8.3

The normalized lowpass filter with a cutoff frequency of 1 rad/s is given as

$$H_P(s) = \frac{1}{s+1}$$

1. Use HP(s) and the BLT to obtain a corresponding IIR digital high-pass filter with a cutoff frequency of 30 Hz, assuming a sampling rate of 200 Hz.
2. Use MATLAB to plot the magnitude and phase frequency responses of H(z)

### solution

1. $\omega_{zp} = 2\pi \times 30$ rad/s, $f_s = 200$ Hz

$$\omega_{sp} = 2f_s \tan(\frac{\omega_{zp}}{2f_s}) = 203.8102$$

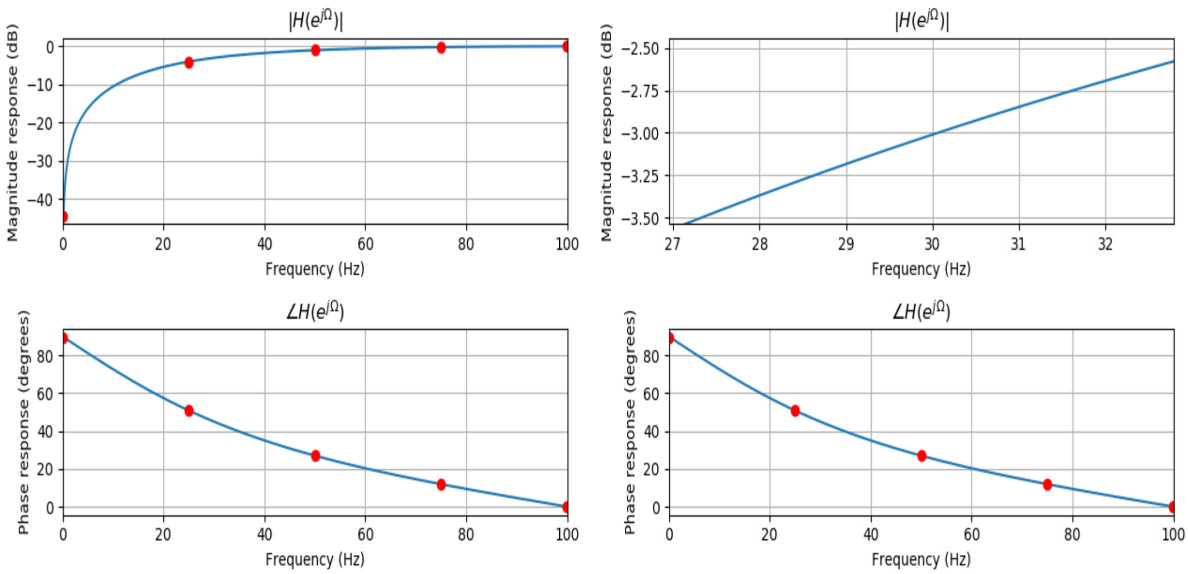
Then substitute $s' = \frac{\omega_{sp}}{s}$, high-pass filter

$$H(s) = \frac{1}{s'+1}|_{s'=\frac{\omega_{sp}}{s}} = \frac{s}{s+203.8101}$$

Then with BLT

$$H(z) = H(s)|_{z=2f_s\frac{1-z^{-1}}{1+z^{-1}}} = \frac{0.6625 - 0.6625z^{-1}}{1-0.3249z^{-1}}$$

2. Plot the magnitude and phase frequency responses of H(z)



## Problem 8.6

Design a first-order digital lowpass Butterworth filter with a cutoff frequency of 1.5 kHz and a passband ripple of 3 dB at a sampling frequency of 8000 Hz.

1. Determine the transfer function and difference equation.
2. Use MATLAB to plot the magnitude and phase frequency responses.

### solution

1. $\omega_{zp} = 2\pi \times 1.5 \times 10^3$ rad/s, $f_s = 8000$ Hz

$$\omega_{sp} = 2f_s \tan(\frac{\omega_{zp}}{2f_s}) = 10690.8582$$

The first-order digital lowpass Butterworth filter

$$H(s') = \frac{1}{s'+1}$$

Then substitute $s' = \frac{s}{\omega_{sp}}$, low-pass filter

$$H(s) = \frac{1}{s'+1}|_{s'=\frac{\omega_{sp}}{s}} = \frac{10690.8582}{s + 10690.8582}$$

Then with BLT, transfer function

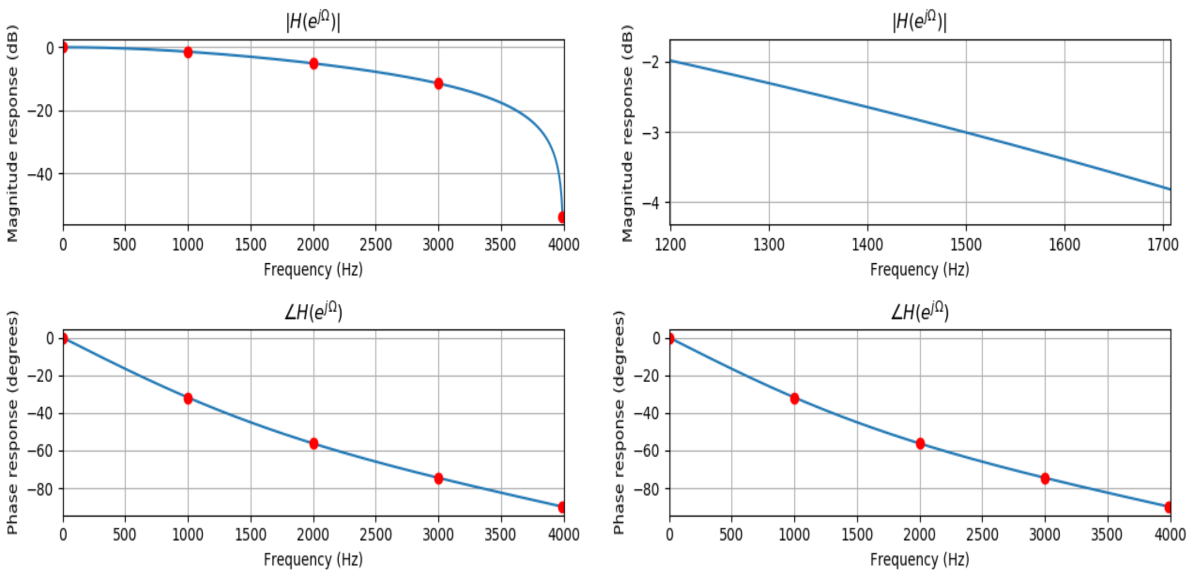
$$H(z) = H(s)|_{z=2f_s\frac{1-z^{-1}}{1+z^{-1}}} = \frac{0.4005 + 0.4005z^{-1}}{1 - 0.1989z^{-1}}$$

The difference function

$$[1 - 0.1989z^{-1}]Y(z) = [0.4005 + 0.4005z^{-1}]X(z)$$
$$y(n) - 0.1989y(n-1) = 0.4005x(n) + 0.4005x(n-1)$$

Thus, $y(n) = 0.4005x(n) + 0.4005x(n-1) + 0.1989y(n-1)$

2. Plot the magnitude and phase frequency responses of H(z)



## Problem 8.9

Design a second-order digital bandpass Butterworth filter with a lower cutoff frequency of 1.9 kHz, an upper cutoff frequency 2.1 kHz, and a passband ripple of 3 dB at a sampling frequency of 8000 Hz.

1. Determine the transfer function and difference equation.
2. Use MATLAB to plot the magnitude and phase frequency responses.

### solution

1. $\omega_{zp} = 2\pi \times [1.9 \times 10^3, 2.1 \times 10^3]$ rad/s, $f_s = 8000$ Hz

$$\omega_{sp} = 2f_s \tan(\frac{\omega_{zp}}{2f_s}) = [14790.2479, 17308.7025]$$

The 2nd-order digital lowpass Butterworth filter

$$H(s') = \frac{1}{s'^2 + 1.4142s' + 1}$$

Then substitute $s' = \frac{s^2+\omega_{sp}[0]\omega_{sp}[1]}{s(\omega_{sp}[1]-\omega_{sp}[0])}$, band-pass filter

$$H(s) = \frac{6.3426 \times 10^6 s^2}{s^4 + 3.5616 \times 10^3 s^3 + 5.1834 \times 10^8 s^2 + 9.1178 \times 10^{11} s + 6.5536 \times 10^{16}}$$

Then with BLT, transfer function

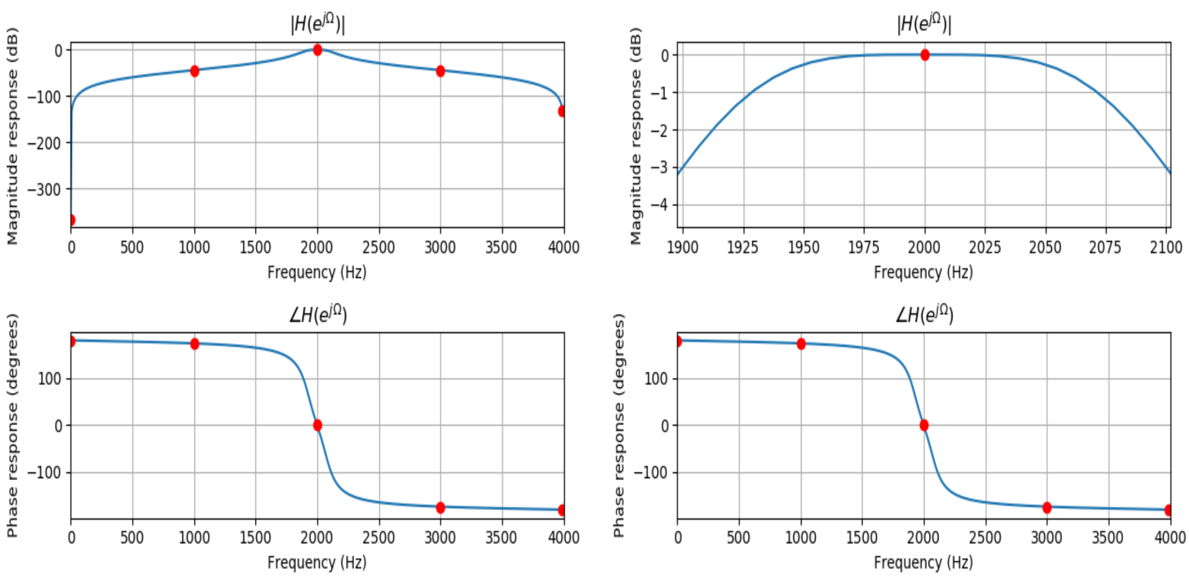
$$H(z) = H(s)|_{z=2f_s \frac{1-z^{-1}}{1+z^{-1}}} = \frac{0.0055 - 0.0111z^{-2} + 0.0055z^{-4}}{1 + 1.7786z^{-2} + 0.8008z^{-4}}$$

The difference function

$$[1 + 1.7786z^{-2} + 0.8008z^{-4}]Y(z) = [0.0055 - 0.0111z^{-2} + 0.0055z^{-4}]X(z)$$
$$y(n) + 1.7786y(n-2) + 0.8008y(n-4) = 0.0055x(n) - 0.0111x(n-2) + 0.0055x(n-4)$$

Thus, $y(n) = 0.0055x(n) - 0.0111x(n-2) + 0.0055x(n-4) - 1.7786y(n-2) - 0.8008y(n-4)$

2. Plot the magnitude and phase frequency responses.



If **order** of Butterworth filter $n = 1$

1. The 1st-order digital lowpass Butterworth filter

$$H(s') = \frac{1}{s' + 1}$$

Then substitute $s' = \frac{s^2 + \omega_{sp}[0]\omega_{sp}[1]}{s(\omega_{sp}[1] - \omega_{sp}[0])}$, band-pass filter

$$H(s) = \frac{2518.4546s}{s^2 + 2518.4546s + 2.56 \times 10^8}$$

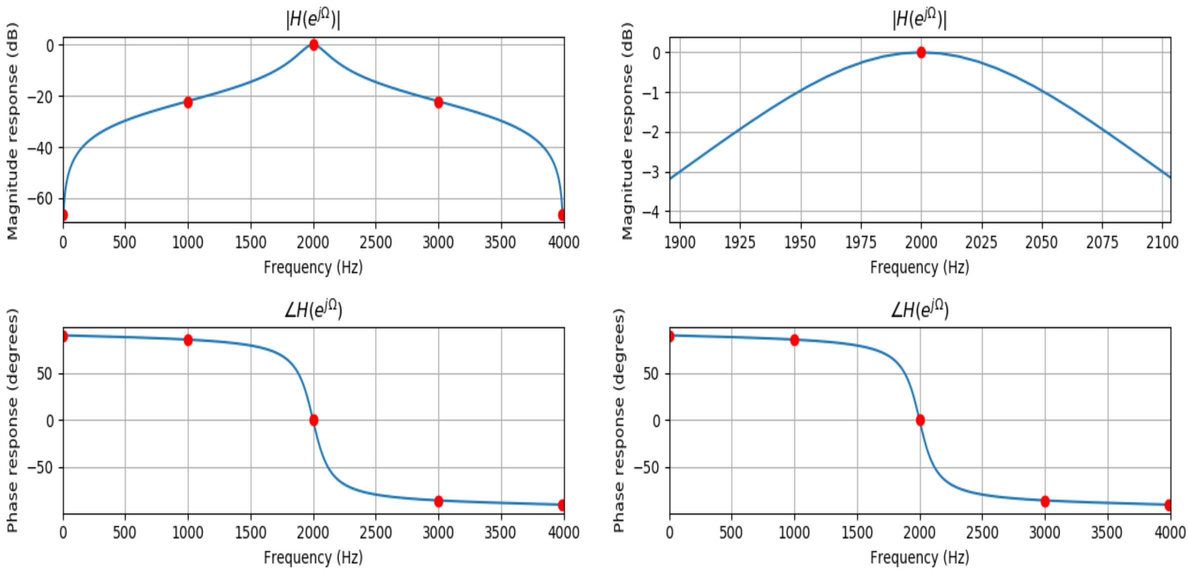
Then with BLT, transfer function

$$H(z) = H(s)|_{z=2f_s \frac{1-z^{-1}}{1+z^{-1}}} = \frac{0.0730 - 0.0730z^{-2}}{1 + 0.8541z^{-2}}$$

The difference equation

$$y(n) = 0.0730x(n) - 0.0730x(n-2) - 0.8541y(n-2)$$

2. Plot the magnitude and phase frequency responses.



## Problem 8.15

Design a second-order bandstop digital Chebyshev filter with the following specifications:

- Center frequency of 2.5 kHz
- Bandwidth of 200 Hz
- 1-dB ripple on stopband (**should be passband**)
- Sampling frequency of 8000 Hz.

1. Determine the transfer function and difference equation.
2. Use MATLAB to plot the magnitude and phase frequency responses.

### solution

1. Determine the transfer function and difference equation.

Because $\frac{\pi}{2}\frac{f_{center}}{f_s/2} = 2\pi\frac{f_{center}}{2f_s} > \frac{\pi}{2}\frac{1}{2} = \frac{\pi}{4}$, and $\tan^2(x) < \tan(x-\Delta x)\tan(x+\Delta x)$ for $x > \frac{\pi}{4}$

For wilder passband, that means attenuation at 2.4kHz, 2.6kHz should < 1dB

<=> bandwidth < 200 Hz

Here we can find $\Delta x' < \Delta x$, satisfy

$$\tan^2(x) = \tan(x-\Delta x)\tan(x+\Delta x')$$

Then, the bandwidth $\frac{2f_s}{2\pi}[\Delta x + \Delta x'] < \frac{2f_s}{2\pi}[2\Delta x] = 200$ Hz

$$\omega_L = 2f_s\tan(x-\Delta x) = 22022.1107$$
$$\omega_H = 2f_s\tan(x+\Delta x') = \frac{[2f_s\tan(x)]^2}{2f_s\tan(x-\Delta x)} = \frac{\omega_0^2}{\omega_L} = 26037.2942$$

Here, $x = 2\pi\frac{f_{center}}{2f_s} = x = 2\pi\frac{2500}{2f_s}, \Delta x = 2\pi\frac{W/2}{2f_s} = 2\pi\frac{100}{2f_s}$

Design Chebyshev filter, $A_p = 1dB, \varepsilon = \sqrt{10^{A_p/10}-1} = 0.5088$

Now with order $n = 2, \varepsilon = 0.5088$, design the Chebyshev filter, n is even

$$H(s') = \frac{1}{\varepsilon 2^{n-1}}\frac{1}{\prod_{m=0}^{\left(\frac{n}{2}\right)-1}\left(s'^2 + [2\times \mathrm{sh}\times s(m)]s' + \left[\mathrm{sh}^2+1-s^2(m)\right]\right)}$$
$$= \frac{0.9826}{s^2+1.0977s+1.1025}$$

Here, $\mathrm{sh} \equiv \sinh\left(\frac{1}{n}\mathrm{arsinh}\left(\frac{1}{\varepsilon}\right)\right), s(m) \equiv \sin\left(\frac{\pi}{2}\left(\frac{1}{n}\right)+\pi\left(\frac{m}{n}\right)\right)$

Then substitute $s' = \frac{s(\omega_H-\omega_L)}{s^2+\omega_H\omega_L}$, band-stop filter

$$H(s) = \frac{0.8913s^4 + 1.0221\times 10^9 s^2 + 2.9303\times 10^{17}}{s^4 + 3.9978\times 10^3 s^3 + 1.1614\times 10^9 s^2 + 2.2923\times 10^{12} s + 3.2878\times 10^{17}}$$

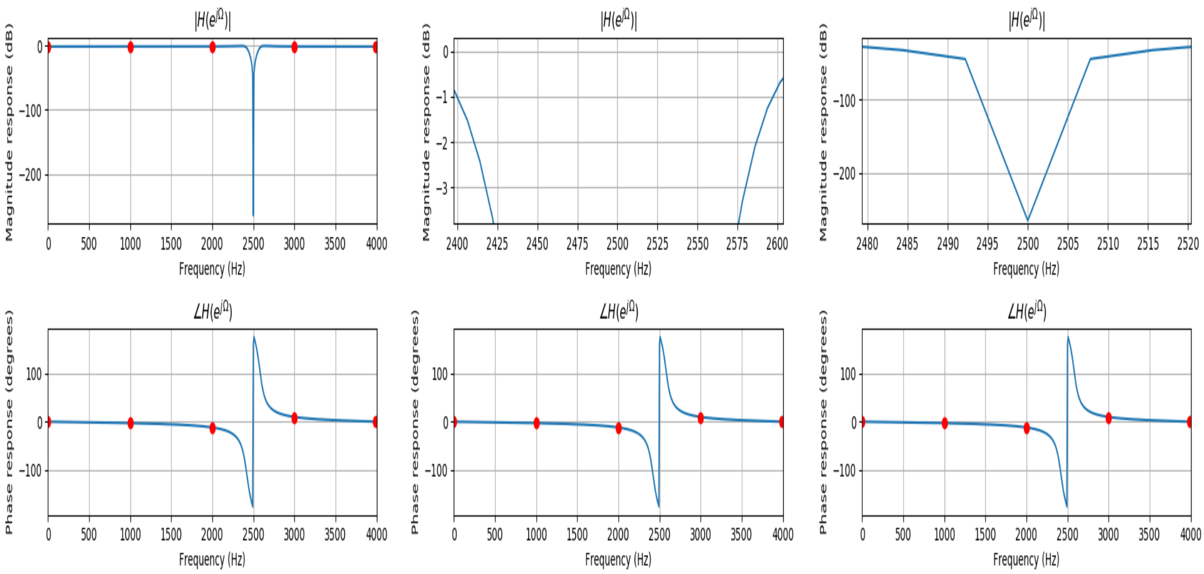
Next with BLT $H(z) \equiv H(s)|_{z=2f_s\frac{1-z^{-1}}{1+z^{-1}}}$, transfer function

$$H(z) = \frac{0.8233 + 1.2602z^{-1} + 2.1288z^{-2} + 1.2602z^{-3} + 0.8233z^{-4}}{1 + 1.4685z^{-1} + 2.3785z^{-2} + 1.3595z^{-3} + 0.8575z^{-4}}$$

The difference equation

$$[1 + 1.4685z^{-1} + 2.3785z^{-2} + 1.3595z^{-3} + 0.8575z^{-4}]Y(z)$$
$$= [0.8233 + 1.2602z^{-1} + 2.1288z^{-2} + 1.2602z^{-3} + 0.8233z^{-4}]X(z)$$
$$y(n) = 0.8233x(n) + 1.2602x(n-1) + 2.1288x(n-2) + 1.2602x(n-3) + 0.8233x(n-4)$$
$$-1.4685y(n-1) - 2.3785y(n-2) - 1.3595y(n-3) - 0.8575y(n-4)$$

2. plot the magnitude and phase frequency responses.



If **order** of Chebyshev filter $n = 1$

1. For wilder passband, that means attenuation at 2.4kHz, 2.6kHz should < 1dB

Design Chebyshev filter, $A_p = 1dB, \varepsilon = \sqrt{10^{A_p/10} - 1} = 0.5088$

Now with order $n = 1, \varepsilon = 0.5088$, design the Chebyshev filter, n is odd

$$H(s') = \frac{1}{\varepsilon 2^{n-1}} \frac{1}{(s + \text{sh}) \prod_{m=0}^{(\frac{n-1}{2})-1} (s^2 + [2 \times \text{sh} \times s(m)]s + [\text{sh}^2 + 1 - s^2(m)])}$$
$$= \frac{1.9652}{s + 1.9652}$$

Here, $\text{sh} \equiv \sinh(\frac{1}{n}\text{arsinh}(\frac{1}{\varepsilon})), s(m) \equiv \sin(\frac{\pi}{2}(\frac{1}{n}) + \pi(\frac{m}{n}))$

Then substitute $s' = \frac{s(\omega_H - \omega_L)}{s^2 + \omega_H \omega_L}$, band-stop filter

$$H(s) = \frac{s^2 + 5.7340 \times 10^8}{s^2 + 2043.1146s + 5.7340 \times 10^8}$$

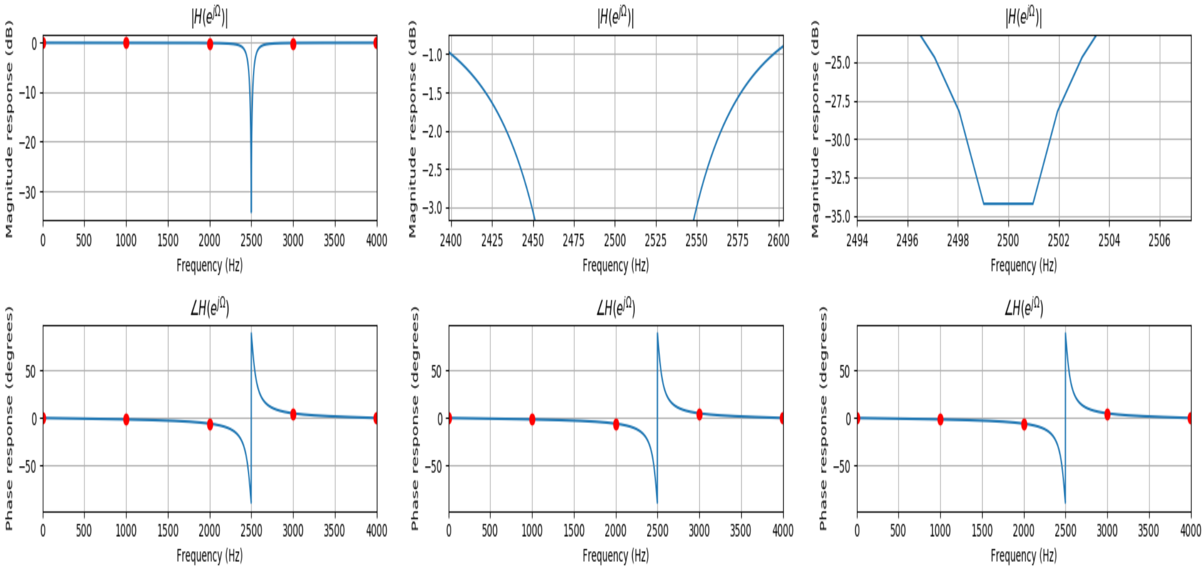
Next with BLT $H(z) \equiv H(s)|_{z=2f_s \frac{1-z^{-1}}{1+z^{-1}}}$, transfer function

$$H(z) = \frac{0.9621 + 0.7363z^{-1} + 0.9621z^{-2}}{1 + 0.7363z^{-1} + 0.9242z^{-2}}$$

The difference equation

$$y(n) = 0.9621x(n) + 0.7363x(n-1) + 0.9621x(n-2) - 0.7363y(n-1) - 0.9242y(n-2)$$

2. plot the magnitude and phase frequency responses.



## Problem 8.19

Consider the following Laplace transfer function:

$$H(s) = \frac{10}{s+10}$$

1. Determine H(z) and the difference equation using the impulse invariant method if the sampling rate fs=10 Hz.
2. Use MATLAB to plot the magnitude frequency response |H(f )| and the phase frequency response ϕ(f ) with respect to H(s) for the frequency range from 0 to fs/2 Hz.
3. Use MATLAB to plot the magnitude frequency response $|H(e^{j\Omega})| = |H(e^{j2\pi fT})|$ and the phase frequency response ϕ(f ) with respect to H(z) for the frequency range from 0 to fs/2Hz.

### solution

1. Laplace inverse transform

$$h(t) = \mathrm{L}^{-1}[H(s)] = 10e^{-10t}$$

Substitute $f_s = 10Hz, t = nT = n/f_s$

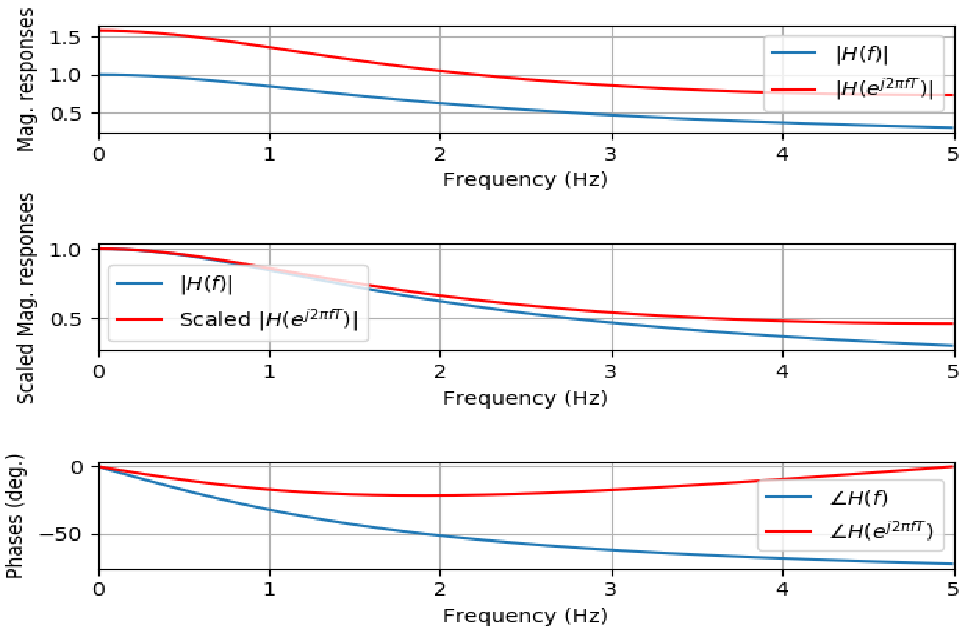
$$T \times h(n) = \frac{1}{f_s} \times h(t)|_{t=nT} = e^{-10(\frac{n}{10})} = e^{-n}$$

Z transform, H(z)

$$H(z) = \mathrm{Z}[T \times h(n)] = \sum_{n=0}^{\infty} e^{-n} z^{-n} = \frac{1}{1 - e^{-1}z^{-1}} = \frac{1}{1 - 0.3679z^{-1}}$$

The difference equation: $y(n) = x(n) + 0.3679y(n-1)$

2. Plot the magnitude and the phase frequency response with respect to H(s), H(z) for the frequency range from 0 to fs/2 Hz.



## Problem 8.23

A second-order notch filter is required to satisfy the following specifications:

- Sampling rate = 8000 Hz
- 3-dB bandwidth: BW=200 Hz
- Narrow passband centered at $f_0$=1000 Hz.

Find the transfer function and difference equation by the pole-zero placement method.

### solution

Set H(z)

$$
\begin{aligned}
&K\frac{(z-e^{j\theta_0})(z+e^{-j\theta_0})}{(z-r_0e^{j\theta_0})(z-r_0e^{-j\theta_0})}\\
&=K\frac{z^2-\cos(\theta_0)z+1}{z^2-2r_0\cos(\theta_0)z+r_0^2}
\end{aligned}
$$

First, calculate $r_0$ with approximate formula

$$r_0=2\pi\times\frac{0.5BW}{f_s}=0.92146$$

compute the $\theta_0$

$$\theta_0=2\pi\frac{f_0}{f_s}=0.7854$$

Then, compute K

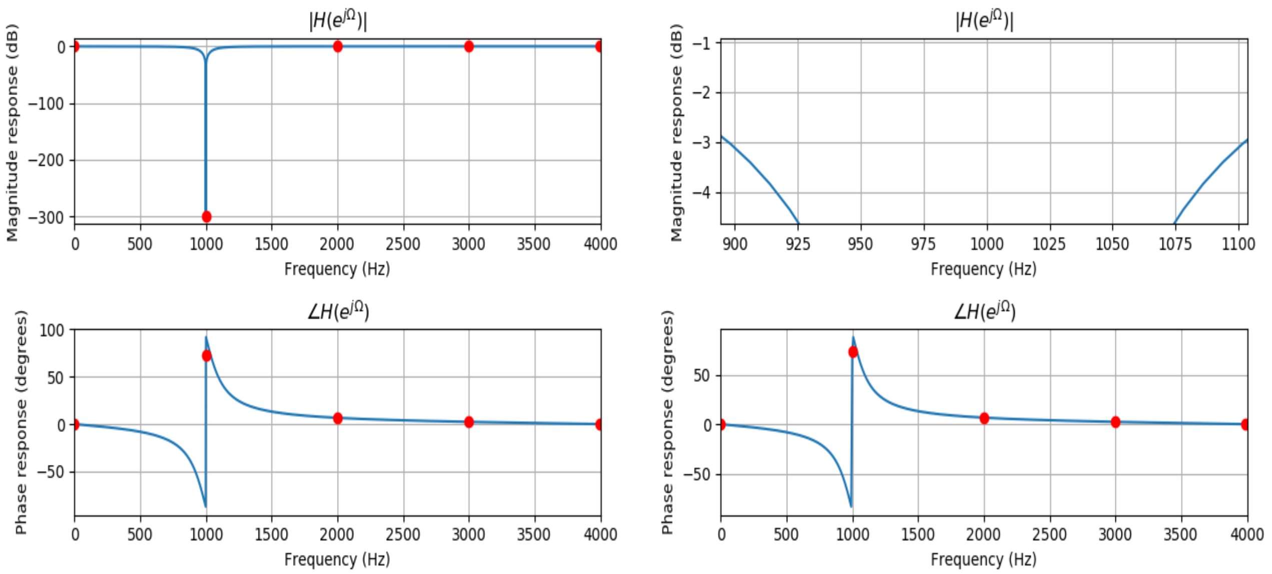
$$K=\frac{(1+r_0^2)-2r_0\cos(\theta_0)}{2-2\cos(\theta_0)}=0.9320$$

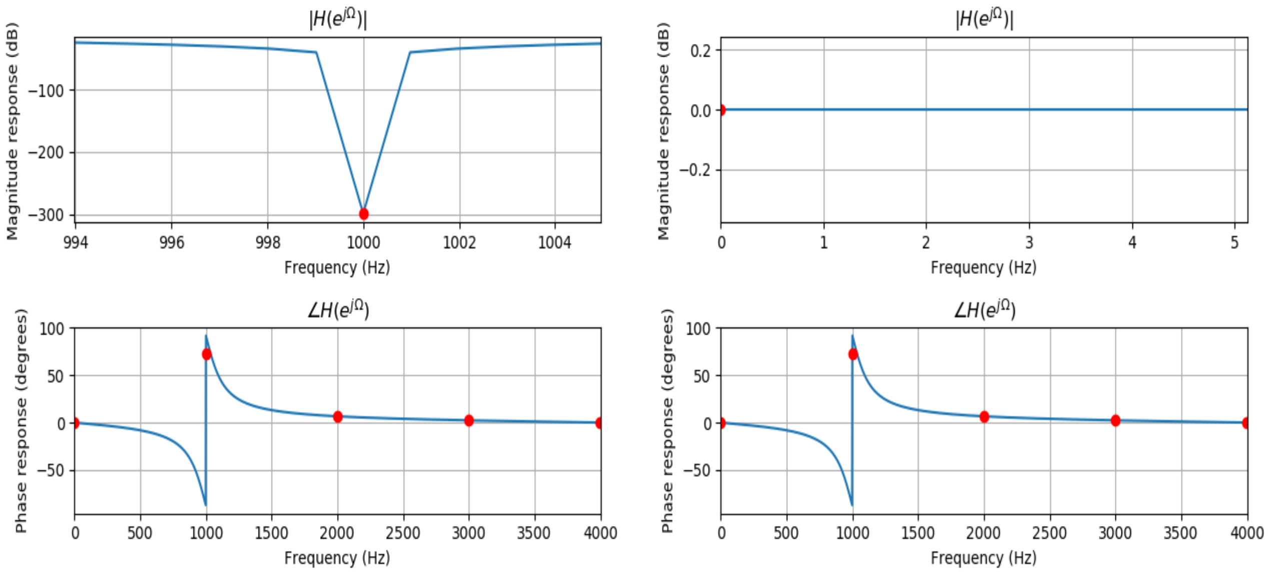
Thus, transfer function

$$H(z) = \frac{0.9320 - 1.3180z^{-1} + 0.9320z^{-2}}{1 - 1.3031z^{-1} + 0.8491z^{-2}}$$

The difference equation

$$y(n) = 0.9320x(n) - 1.3180x(n-1) + 0.9320x(n-2) + 1.3031y(n-1) - 0.8491x(n-2)$$



## Problem 8.25

A first-order lowpass filter is required to satisfy the following specifications:

- Sampling rate = 8000 Hz
- 3-dB cutoff frequency: $f_c$ = 3800 Hz
- Zero gain at 4000 Hz.

Find the transfer function and difference equation by the pole-zero placement method.

### solution

Set H(z)

$$H(z) = K\frac{z+1}{z-\alpha}$$

First, compute $\alpha$

$$\alpha = \frac{1-\tan(\pi\frac{f_c}{f_s})}{1+\tan(\pi\frac{f_c}{f_s})} = -0.8541$$

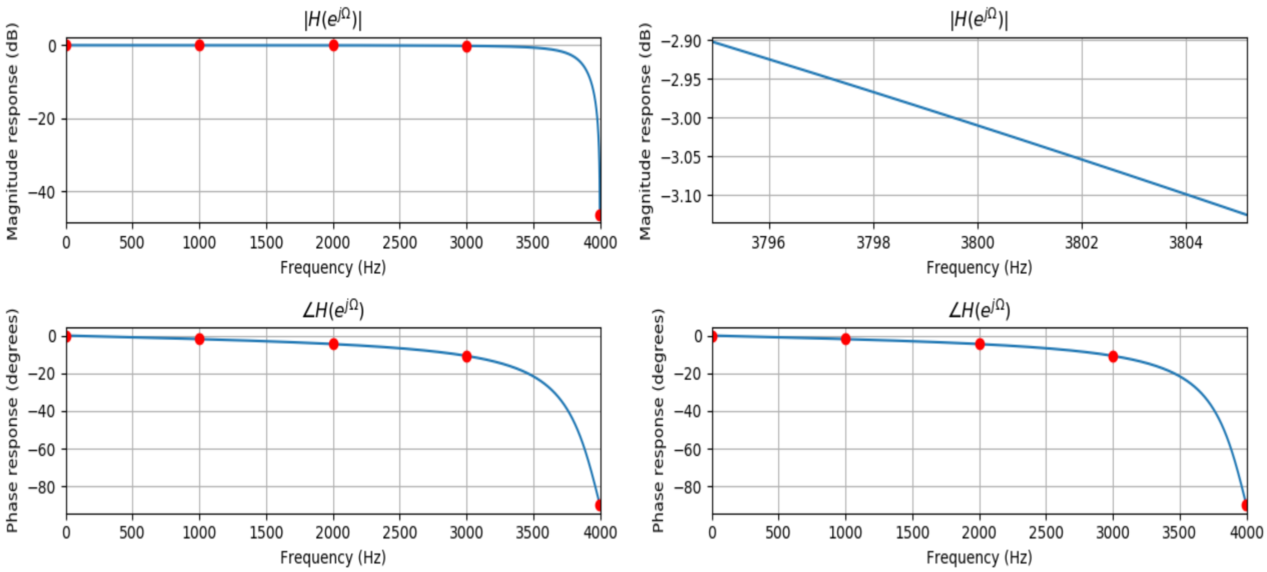
Then compute K

$$K = \frac{|1-\alpha|}{2} = 0.9270$$

Thus, transfer function

$$H(z) = \frac{0.9270 + 0.9270z^{-1}}{1+0.8541z^{-1}}$$

The difference equation $y(n) = 0.9270x(n) + 0.9270x(n-1) - 0.8541y(n-1)$



## Problem 8.26

A first-order highpass filter is required to satisfy the following specifications:

- Sampling rate = 8000 Hz
- 3-dB cutoff frequency: $f_c$= 3850 Hz
- Zero gain at 0 Hz.

Find the transfer function and difference equation by the pole-zero placement method.

### solution

Set H(z)

$$H(z) = K\frac{z-1}{z-\alpha}$$

First, compute $\alpha$

$$\alpha = \frac{1-\tan(\pi\frac{f_c}{f_s})}{1+\tan(\pi\frac{f_c}{f_s})} = -0.8886$$

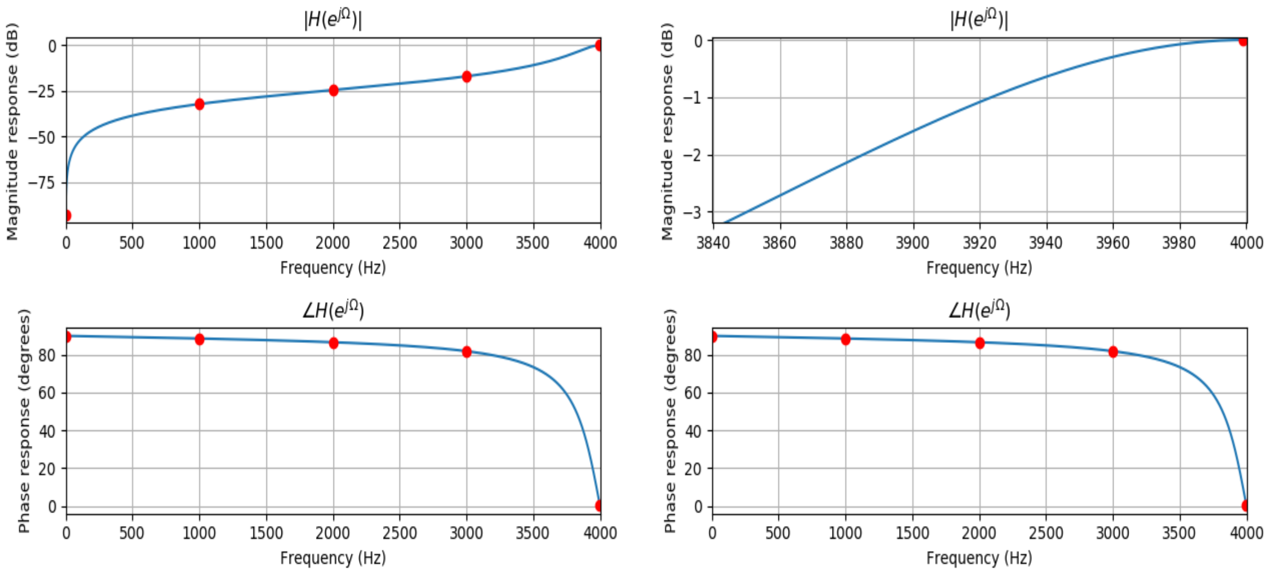
Then compute K

$$K = \frac{|1+\alpha|}{2} = 0.0557$$

Thus, transfer function

$$H(z) = \frac{0.0557 - 0.0557z^{-1}}{1+0.8886z^{-1}}$$

The difference equation $y(n) = 0.0557x(n) - 0.0557x(n-1) - 0.8886y(n-1)$



# MATLAB Projects

## Problem 8.52

**Digital speech and audio equalizer** Design a seven-band audio equalizer using fourth-order bandpass filters with a sampling rate of 44.1 kHz. The center frequencies are listed in **Table 8.14**. In this project, use the designed equalizer to process a stereo audio (“No9seg.wav”).

- Plot the magnitude response for each filter bank.
- Listen and evaluate the processed audio with the following gain settings:

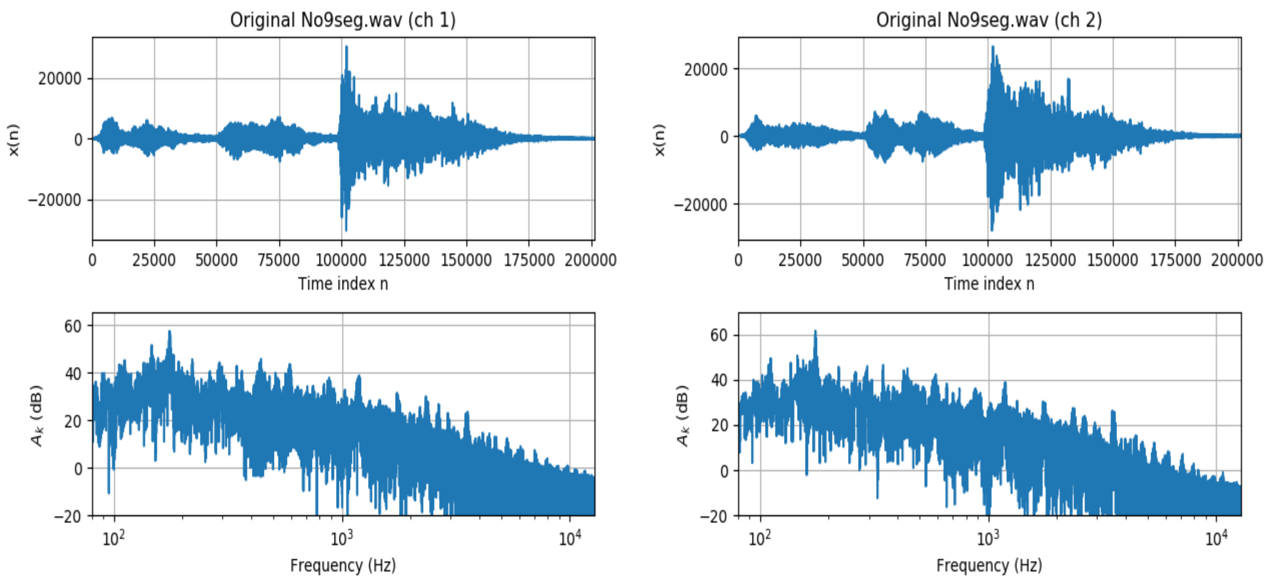
1. each filter bank gain=0 (no equalization)
2. low-pass filtered
3. band-pass filtered
4. high-pass filtered

**Table 8.14** Specification for Center Frequencies and Bandwidths

| **Center Frequency (Hz)** | **160** | **320** | **640** | **1280** | **2560** | **5120** | **10,240** |
|---|---|---|---|---|---|---|---|
| Bandwidth (Hz) | 80 | 160 | 320 | 640 | 1280 | 2560 | 5120 |

### solution

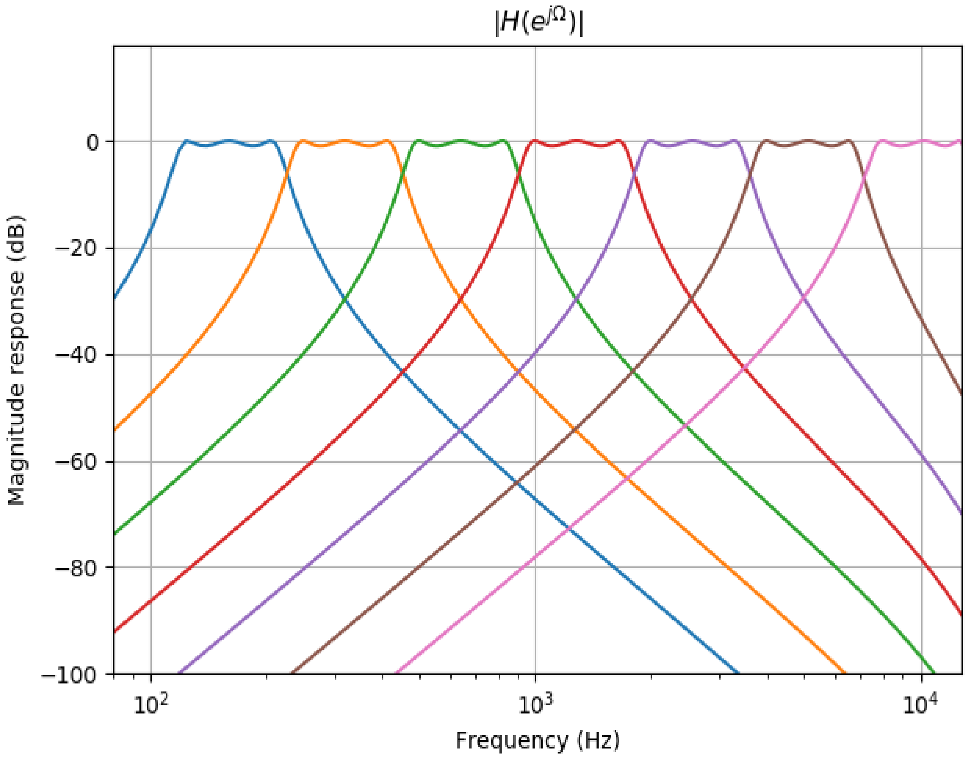
#### Original signal



**Band-pass filter**

Here we design the filter:

- Chebyshev I filter: order $n = 3$
- Pass band $A_p = 1$ dB
- Filter type: band-pass
- center frequency $f_c = [160, 320, 640, 1280, 2560, 5120, 10240]$
- Band-width $BW = [80, 160, 320, 640, 1280, 2560, 5120]$
- Sampling frequency $f_s = 44.1$ kHz




Design Chebyshev filter, $A_p = 1dB, \varepsilon = \sqrt{10^{A_p/10} - 1} = 0.5088$

Now with order $n = 3, \varepsilon = 0.5088$, design the Chebyshev filter, n is odd

$$H(s') = \frac{1}{\varepsilon 2^{n-1}} \frac{1}{(s + \text{sh}) \prod_{m=0}^{(\frac{n-1}{2})-1} \left(s^2 + [2 \times \text{sh} \times s(m)]s + [\text{sh}^2 + 1 - s^2(m)]\right)}$$
$$= 0.4913 \times \frac{1}{s + 0.4942} \times \frac{1}{s^2 + 0.4942s + 0.9942}$$

Here, $\text{sh} \equiv \sinh\left(\frac{1}{n}\text{arsinh}\left(\frac{1}{\varepsilon}\right)\right), s(m) \equiv \sin\left(\frac{\pi}{2}\left(\frac{1}{n}\right) + \pi\left(\frac{m}{n}\right)\right)$

Then substitute $s' = \frac{s(\omega_H - \omega_L)}{s^2 + \omega_H \omega_L}$, band-stop filter

Here with $\omega = (2f_s) \times \tan(\pi \frac{f}{f_s}) = (2f_s) \times \tan(\frac{2\pi f}{2f_s})$,

we have $\omega_L = [754.0, 1508.11, 3017.1, 6041.28, 12139.51, 24747.84, 53725.46]$

and $\omega_H = [1340.5, 2681.49, 5367.0, 10766.21, 21794.31, 45827.05, 115719.83]$

$$
\begin{aligned}
&H(s)\\
&= 0.4913 \times \frac{586.4957s}{1.0s^2 + 289.8289s + 1010735.0299}\\
&\times \frac{343977.2177s^2}{s^4 + 289.8289s^3 + 2363453.7874s^2 + 292940262.0727s + 1021585300762.115} && [1st]\\
&= 0.4913 \times \frac{1173.382s}{1.0s^2 + 579.8509s + 4043990.9005}\\
&\times \frac{1376825.3819s^2}{s^4 + 579.8509s^3 + 9456827.9109s^2 + 2344911788.4637s + 16353862403450.79} && [2nd]\\
&= 0.4913 \times \frac{2349.8925s}{1.0s^2 + 1161.2478s + 16192794.6791}\\
&\times \frac{5521994.934s^2}{s^4 + 1161.2478s^3 + 37875582.0497s^2 + 18803847462.5707s + 2.6221 \times 10^{14}} && [3rd]\\
&= 0.4913 \times \frac{4724.9293s}{s^2 + 2334.9212s + 65041670.5048}\\
&\times \frac{22324957.049s^2}{s^4 + 2334.9212s^3 + 152278915.7077s^2 + 151867173966.5871s + 4.2304 \times 10^{15}} && [4th]\\
&= 0.4913 \times \frac{9654.8014s}{s^2 + 4771.119s + 264572327.7615}\\
&\times \frac{93215190.0733s^2}{s^4 + 4771.119s^3 + 621819625.0523s^2 + 1262306072671.3513s + 6.9999 \times 10^{16}} && [5th]\\
&= 0.4913 \times \frac{21079.2088s}{s^2 + 10416.7253s + 1134120477.0311}\\
&\times \frac{444333041.5525s^2}{1.0s^4 + 10416.7253s^3 + 2709998902.0356s^2 + 11813821511707.322s + 1.2862 \times 10^{18}} && [6th]\\
&= 0.4913 \times \frac{61994.3763s}{s^2 + 30635.7984s + 6217101105.0179}\\
&\times \frac{3843302690.7742s^2}{1.0s^4 + 30635.7984s^3 + 16255231373.6225s^2 + 190465856278593.03s + 3.8652 \times 10^{19}} && [7th]
\end{aligned}
$$

Thus, H(z)

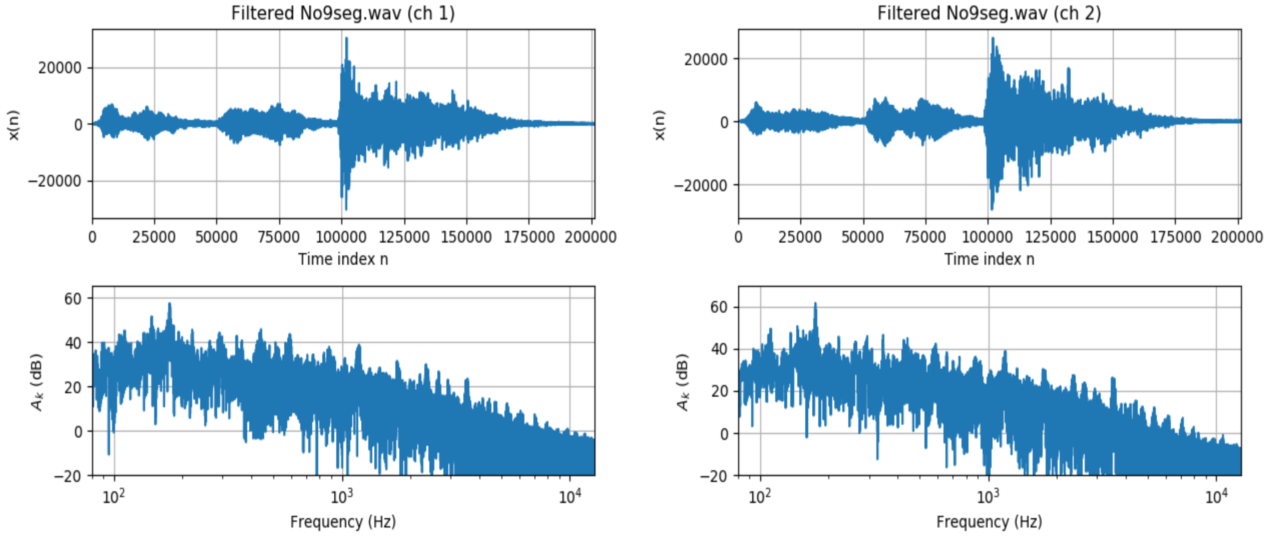
$$
\begin{aligned}
&H(z) \\
&= [(1.4345 \times 10^{-7} - 4.3035 \times 10^{-7} z^{-2} - 0.0z^{-3} + 4.3035 \times 10^{-7} z^{-4} - 1.4345 \times 10^{-7} z^{-6}) \\
&/(1.0 - 5.9852z^{-1} + 14.9276z^{-2} - 19.8588z^{-3} + 14.8624z^{-4} - 5.933z^{-5} + 0.9869z^{-6})] \qquad [1st] \\
&= [(1.1398 \times 10^{-6} - 3.4194 \times 10^{-6} z^{-2} + 0.0z^{-3} + 3.4194 \times 10^{-6} z^{-4} - 1.1398 \times 10^{-6} z^{-6}) \\
&/(1.0 - 5.967z^{-1} + 14.8421z^{-2} - 19.6985z^{-3} + 14.7127z^{-4} - 5.8633z^{-5} + 0.9741z^{-6})] \qquad [2nd] \\
&= [(8.9896 \times 10^{-6} - 2.6969 \times 10^{-5} z^{-2} - 0.0z^{-3} + 2.6969 \times 10^{-5} z^{-4} - 8.9896 \times 10^{-6} z^{-6}) \\
&/(1.0 - 5.9207z^{-1} + 14.6327z^{-2} - 19.3225z^{-3} + 14.3786z^{-4} - 5.7169z^{-5} + 0.9488z^{-6})] \qquad [3rd] \\
&= [(6.9757 \times 10^{-5} - 0.0002093z^{-2} + -0.0z^{-3} + 0.0002093z^{-4} - 6.9757 \times 10^{-5} z^{-6}) \\
&/(1.0 - 5.7893z^{-1} + 14.0683z^{-2} - 18.3659z^{-3} + 13.5846z^{-4} - 5.3981z^{-5} + 0.9004z^{-6})] \qquad [4th] \\
&= [(0.0005209 - 0.001563z^{-2} + 0.0z^{-3} + 0.001563z^{-4} - 0.000521z^{-6}) \\
&/(1.0 - 5.3813z^{-1} + 12.4453z^{-2} - 15.7999z^{-3} + 11.6086z^{-4} - 4.6826z^{-5} + 0.812z^{-6})] \qquad [5th] \\
&= [(0.0035 - 0.0z^{-1} - 0.0106z^{-2} + 0.0106z^{-4} + 0.0z^{-5} - 0.0035z^{-6}) \\
&/(1.0 - 4.0872z^{-1} + 8.1194z^{-2} - 9.5695z^{-3} + 7.0896z^{-4} - 3.1149z^{-5} + 0.6669z^{-6})] \qquad [6th] \\
&= [(0.0183 - 0.0548z^{-2} + 0.0548z^{-4} - 0.0183z^{-6}) \\
&/(1.0 - 0.5511z^{-1} + 2.0431z^{-2} - 0.7891z^{-3} + 1.6473z^{-4} - 0.3362z^{-5} + 0.4822z^{-6})] \qquad [7th]
\end{aligned}
$$

$$Y(z) \equiv X(z) + \sum_{k=1}^{7} \text{Gain}_k \times H(z)_k X(z)$$

$$y(n) = x(n) + \sum_{k=1}^{7} \text{Gain}_k \times [h_k(n) * x(n)]$$

**no equalization**

$\text{Gain} = [0, 0, 0, 0, 0, 0, 0]$



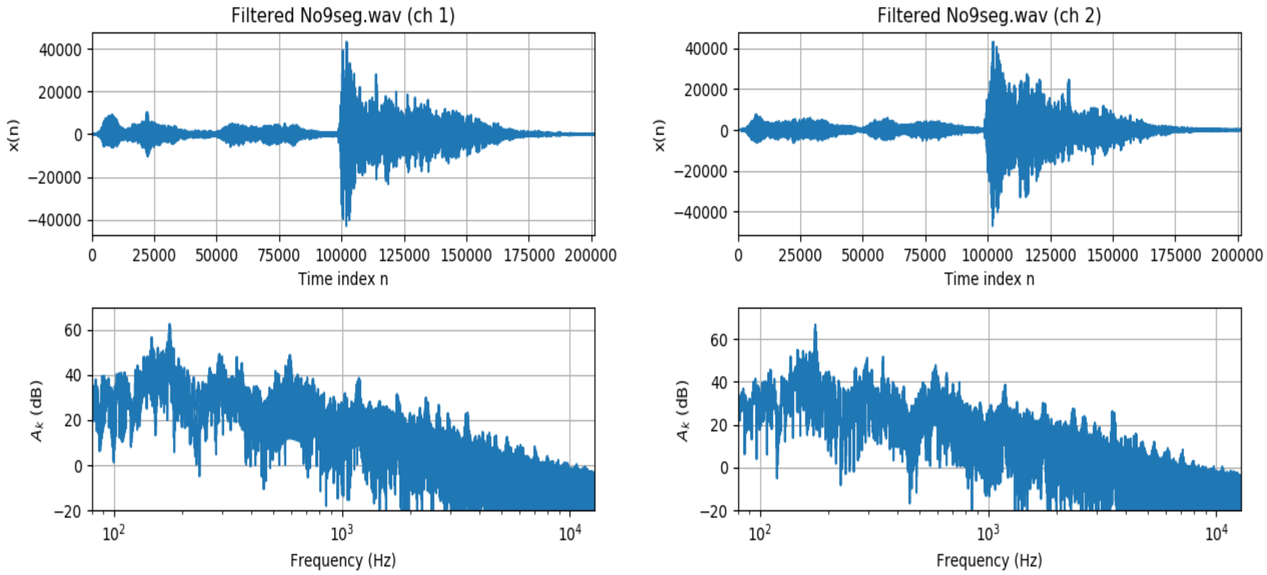

**low-pass filtered**

$\text{Gain} = [1, 1, 1, 0, 0, 0, 0]$

We can see there are 3 peaks [160, 320, 640] in the filtered spectrum,

components in these band [80, 160, 320] are strengthened by the low-pass filter.

We can hear low frequency components more clearly.



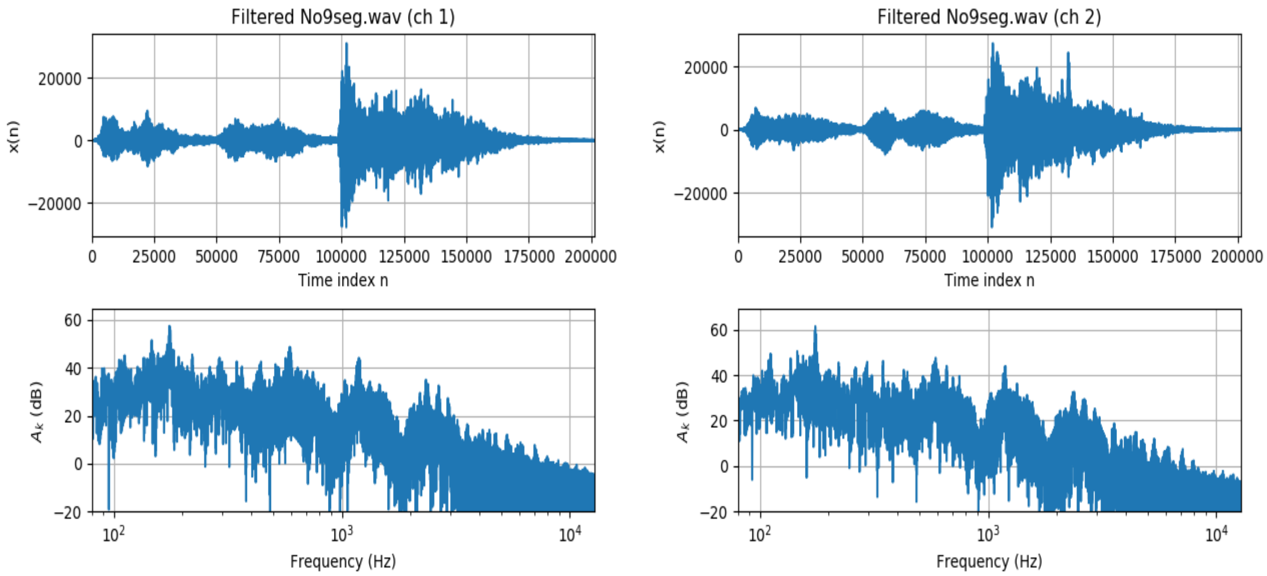
**band-pass filtered**

$$\mathrm{Gain} = [0, 0, 1, 1, 1, 0, 0]$$

We can see there are 3 peaks[640, 1280, 2560] in the filtered spectrum,

components in these band[320, 640, 1280] are strengthened by the band-pass filter.

We can hear the frequency components in specific frequency band more clearly.



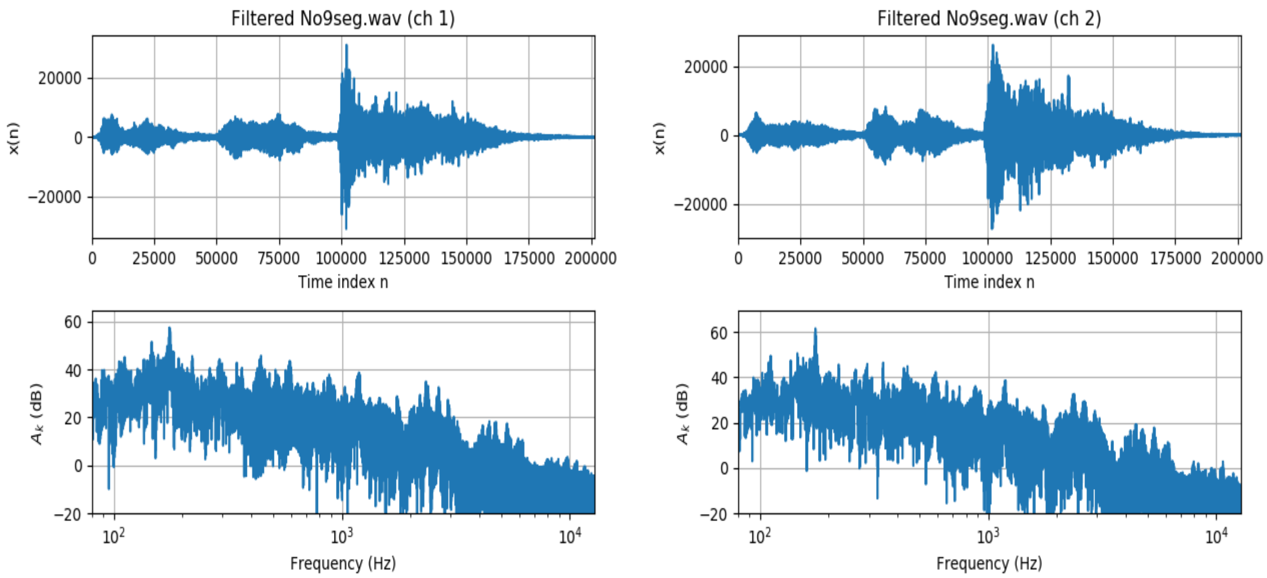

**high-pass filtered**

$$\mathrm{Gain} = [0, 0, 0, 0, 1, 1, 1]$$

We can see there are 3 peaks[2560, 5120, 10240] in the filtered spectrum,

components in these band[1280, 2560, 5120] are strengthened by the high-pass filter.

We can hear low frequency components more clearly.



Here is the main **Python** script with my IIR implementation library.

The **IIR library** implements：

- Calculation, substitution of Polynomial, Fraction of Polynomial $H(s), H(z)$
- BLT, unit low-pass filter $H(s')$ to low-pass, high-pass, band-pass, band-stop
- Magnitude $|H|$, Phase $\angle H$ of $H(s), H(z)$
- FFT to calculate $A_k$ of $X(k), Y(k)$
- Butterworth, Chebyshev I filter: $H(s)$
- IIR filter: $y(n) = \sum_{k=0}^{M} b_k x(n-k) - \sum_{k=1}^{N} a_k y(n-k)$
- Pole-zero Placement parameters
- Plot of Impulse invariance

```
from scipy.io.wavfile import write, read # save sounds
from iir_filter.fft1d import plot_spectrum_dB
from iir_filter.frac import Frac, convert_s2z
from iir_filter.poly import Poly, Polyz
from iir_filter.util import convert_omega_z2s, filter_subs, calc_omega_pass
from iir_filter.calc_mag_angle import calc_mag_angle, plot_mag_freq_multiple
from iir_filter.protype import chebyshev_protype, calc_cheby_eps2
from iir_filter.iir_filter import iir_filter
from math import pi, sqrt, ceil
from functools import reduce

f_sample, list_input = read("./No9seg.wav") # sample rate,  input
list_input_ch1, list_input_ch2 = list_input.T[0], list_input.T[1]
plot_spectrum_dB(list_input_ch1, f_sample, path_fig="../p8_52_input_ch1.png",
str_title="Original No9seg.wav (ch 1)")
plot_spectrum_dB(list_input_ch2, f_sample, path_fig="../p8_52_input_ch2.png",
str_title="Original No9seg.wav (ch 2)")

list_f_center = [160, 320, 640, 1280, 2560, 5120, 10240]
list_BW = [80, 160, 320, 640, 1280, 2560, 5120]
list_omega_pass_low = [2*pi*(f_center - 0.5 * BW) for f_center, BW in
list(zip(list_f_center, list_BW))]
list_omega_pass_high = [2*pi*(f_center + 0.5 * BW) for f_center, BW in
list(zip(list_f_center, list_BW))]
list2D_omega_pass_z = list(zip(list_omega_pass_low, list_omega_pass_high))
num_filter = len(list2D_omega_pass_z)
order = 3
A_p = 1
epsilon = sqrt( calc_cheby_eps2(A_p) )
print("epsilon = " + str(epsilon))
list_H_s = chebyshev_protype(order, epsilon)
print(list_H_s)
list_H_z = []
list2D_mag, list2D_omega = [], []
for ind in range(num_filter):
    list_omega_pass_z = list2D_omega_pass_z[ind]
    list_omega_pass_s = calc_omega_pass(list_omega_pass_z, f_sample,
str_filter_type="band_pass")
    print(list_omega_pass_s)
```

```
    list_H_subs = [filter_subs(H_s, list_omega_pass_s, str_filter_type="band_pass") for
H_s in list_H_s]
    print(list_H_subs)
    H_subs = reduce(lambda x,y: x * y, list_H_subs)
    print(H_subs)
    H_z = convert_s2z(H_subs, f_sample)
    print(H_z)
    list_H_z.append(H_z)
    list_mag, list_angle, list_omega = calc_mag_angle(H_z, num_point=4096)
    list2D_mag.append(list_mag)
    list2D_omega.append(list_omega)
plot_mag_freq_multiple(list2D_mag, list2D_omega, f_sample, path_fig="../p8_52_H_z.png")
# band_gain = [1] + [0, 0, 0, 0, 0, 0, 0] # the first 1 represent original input gain: no
equalization
# band_gain = [1] + [1, 1, 1, 0, 0, 0, 0] # low pass
# band_gain = [1] + [0, 0, 1, 1, 1, 0, 0] # band pass
band_gain = [1] + [0, 0, 0, 0, 1, 1, 1] # high pass
list2D_output_ch1 = [list_input_ch1]
list2D_output_ch2 = [list_input_ch2]
for H_z in list_H_z:
    list2D_output_ch1.append( iir_filter(list_input_ch1, H_z) )
    list2D_output_ch2.append( iir_filter(list_input_ch2, H_z) )
list2D_output_ch1 = list(map(list, zip(*list2D_output_ch1))) # transpose
list2D_output_ch2 = list(map(list, zip(*list2D_output_ch2)))
list_output_ch1, list_output_ch2 = [], []
for out_ch1, out_ch2 in list(zip(list2D_output_ch1, list2D_output_ch2)):
    list_output_ch1.append( int(sum([elem * gain for elem, gain in list(zip(out_ch1,
band_gain))])) )
    list_output_ch2.append( int(sum([elem * gain for elem, gain in list(zip(out_ch2,
band_gain))])) )
plot_spectrum_dB(list_output_ch1, f_sample, path_fig="../p8_52_output_high pass_ch1.png",
str_title="Filtered No9seg.wav (ch 1)")
plot_spectrum_dB(list_output_ch2, f_sample, path_fig="../p8_52_output_high pass_ch2.png",
str_title="Filtered No9seg.wav (ch 2)")
import numpy as np
list_output = np.asarray([list_output_ch1, list_output_ch2]).T
max_output = max(np.max(list_output), -np.min(list_output))
factor = (2**(16-1)/max_output)
list_output_scaled = np.floor(list_output * factor).astype(np.int16)
write("../No9seg_high pass.wav", f_sample, list_output_scaled)
```