Exam 2

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Problem 1

Given the following DSP system with a sampling rate of 8000 Hz

$$y(n) = 0.5x(n) + 0.5y(n-2)$$

a. Obtain transfer function $H(z)$

b. Obtain the frequency response $H\left(e^{j\Omega}\right)$ and then the magnitude response $|H\left(e^{j\Omega}\right)|$

c. Compute the filter gain at the frequency of 0 Hz, 1000 Hz, 2000 Hz, 3000Hz, 4000 Hz, respectively.

d. Make a plot of the magnitude frequency response.

e. Determine the filter type: that is, the lowpass filter, or highpass filter, or bandpass filter, or bandstop filter.

(20 points)

solution

a. transfer function $H(z)$

$$(1 - 0.5z^{-2})Y(z) = 0.5X(z)$$
$$H(z) \equiv \frac{Y(z)}{X(z)} = \frac{0.5}{1 - 0.5z^{-2}}$$

b. $H\left(e^{j\Omega}\right)$ and magnitude response $|H\left(e^{j\Omega}\right)|$

$$H(e^{j\Omega}) = H(z)|_{z=e^{j\Omega}} = \frac{0.5}{1 - 0.5\cos(2\Omega) + j0.5\sin(2\Omega)}$$
$$|H\left(e^{j\Omega}\right)| = \frac{0.5}{\sqrt{(1-0.5\cos(2\Omega))^2 + (0.5\sin(2\Omega))^2}} = \frac{0.5}{\sqrt{1.25 - \cos(2\Omega)}}$$

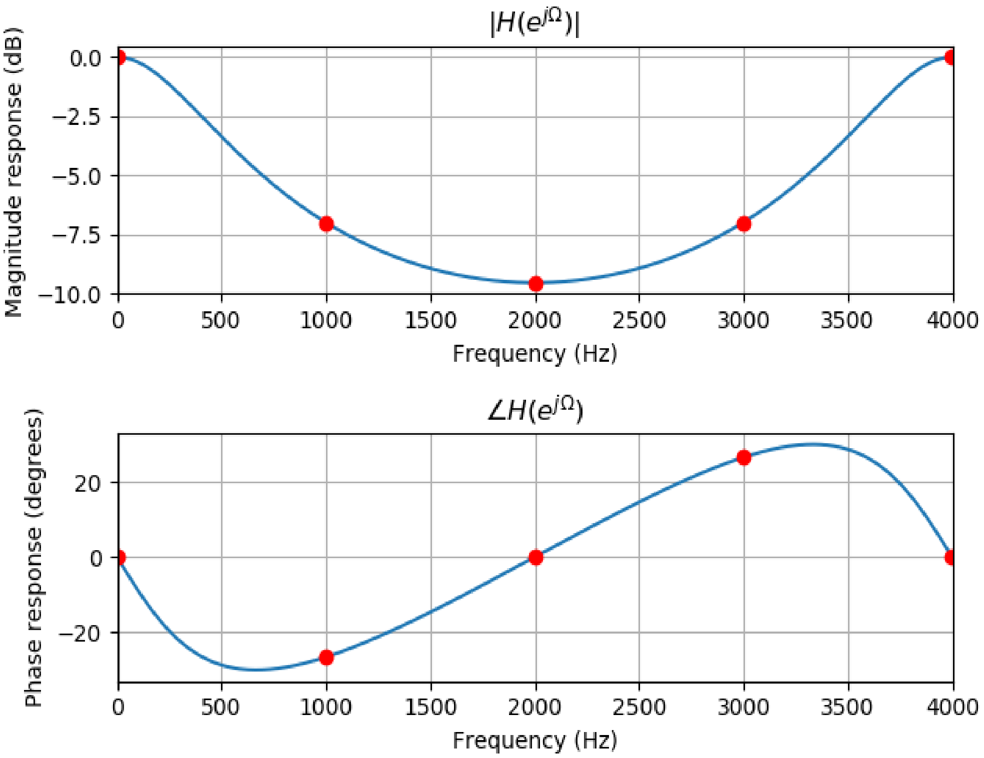
c. Compute the filter gain at f=[0, 1000, 2000, 3000, 4000] Hz

$$f = [0, 1000, 2000, 3000, 4000]Hz \Longrightarrow \Omega = 2\pi\frac{f}{f_s} = [0, \pi/4, \pi/2, 3\pi/4, \pi]$$

Then the filter gain

$$|H\left(e^{j\Omega}\right)| = [\frac{0.5}{\sqrt{0.25}}, \frac{0.5}{\sqrt{1.25}}, \frac{0.5}{\sqrt{2.25}}, \frac{0.5}{\sqrt{1.25}}, \frac{0.5}{\sqrt{0.25}}] = [1.0, 0.4472, 0.3333, 0.4472, 1.0]$$

d. Plot of the magnitude frequency response.



e. Filter type:

Bandstop Filter

Problem 2

Design a 5-tap band-stop FIR filter whose lower cutoff frequency is 800 Hz and upper cut-off frequency is 1200 Hz using the Fourier transform method with Hamming window. Assume the sampling frequency is 8000 Hz.

a. List the FIR filter coefficients

b. Determine the transfer function

c. Determine the DSP equation

d. Set up MATLAB routine “freqz()” to obtain the frequency response plot

e. Draw the realization block diagram for linear phase implementation

(25 points)

solution

a. List the FIR filter coefficients

$$\Omega_L = 2\pi \frac{f_L}{f_s} = 0.2\pi, \Omega_H = 2\pi \frac{f_H}{f_s} = 0.3\pi$$

For band-stop filter, M = (5-1)/2 = 2

$$h(n) = \begin{cases} \frac{\pi - \Omega_H + \Omega_L}{\pi} & \text{for } n = 0 \\ -\frac{\sin(\Omega_H n)}{n\pi} + \frac{\sin(\Omega_L n)}{n\pi} & \text{for } n \neq 0 \quad -M \leq n \leq M \end{cases}$$

Then

$$h(n) = [h(-M), \ldots, h(M)] = [0, -0.07042, 0.9, -0.07042, 0]$$

Thus

$$w_{\text{hamm}}(n) = 0.54 + 0.46 \cos\left(\frac{n\pi}{M}\right), -M \leq n \leq M = [0.08, 0.54, 1, 0.54, 0.08]$$
$$h_w(n) = h(n) \cdot w_{\text{hamm}}(n) = [0, -0.03803, 0.9, -0.03803, 0]$$

b. Determine the transfer function

$$H(z) = \sum_{n=-M}^{M} h_w(n) \cdot z^{-n+M} = -0.03803 z^{-1} + 0.9 z^{-2} - 0.03803 z^{-3}$$

c. Determine the DSP equation

$$y(n) = -0.03803 x(n-1) + 0.9 x(n-2) - 0.03803 x(n-3)$$

d. Set up MATLAB routine “freqz()” to obtain the frequency response plot

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freqz([0, -0.03803, 0.9, -0.03803, 0], [1], 4096, fs)
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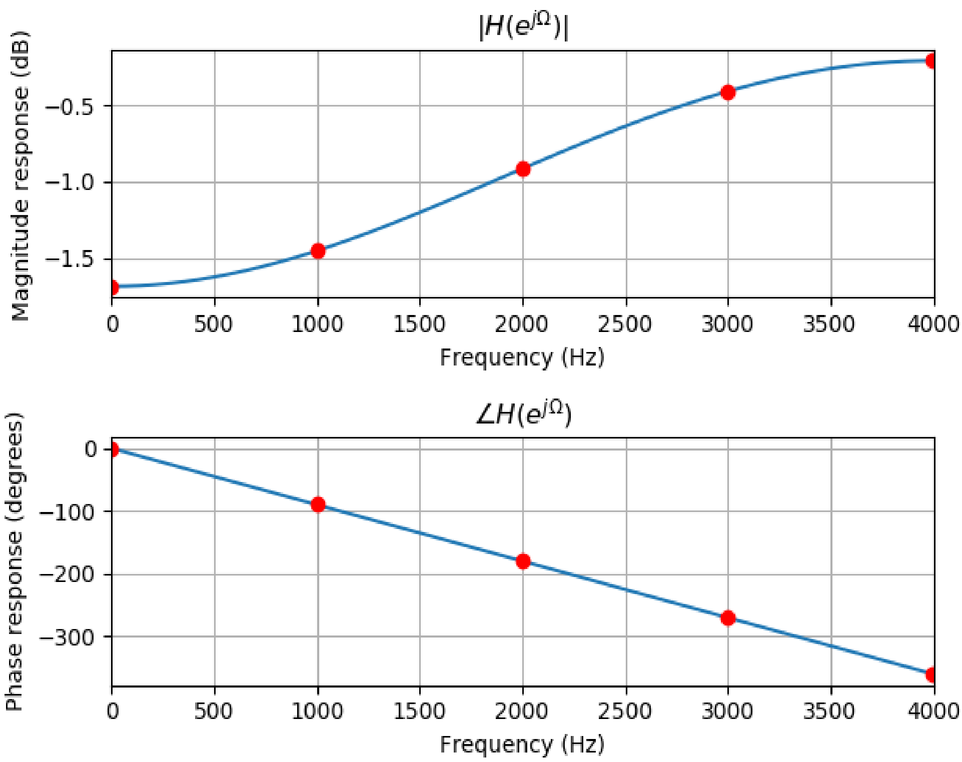
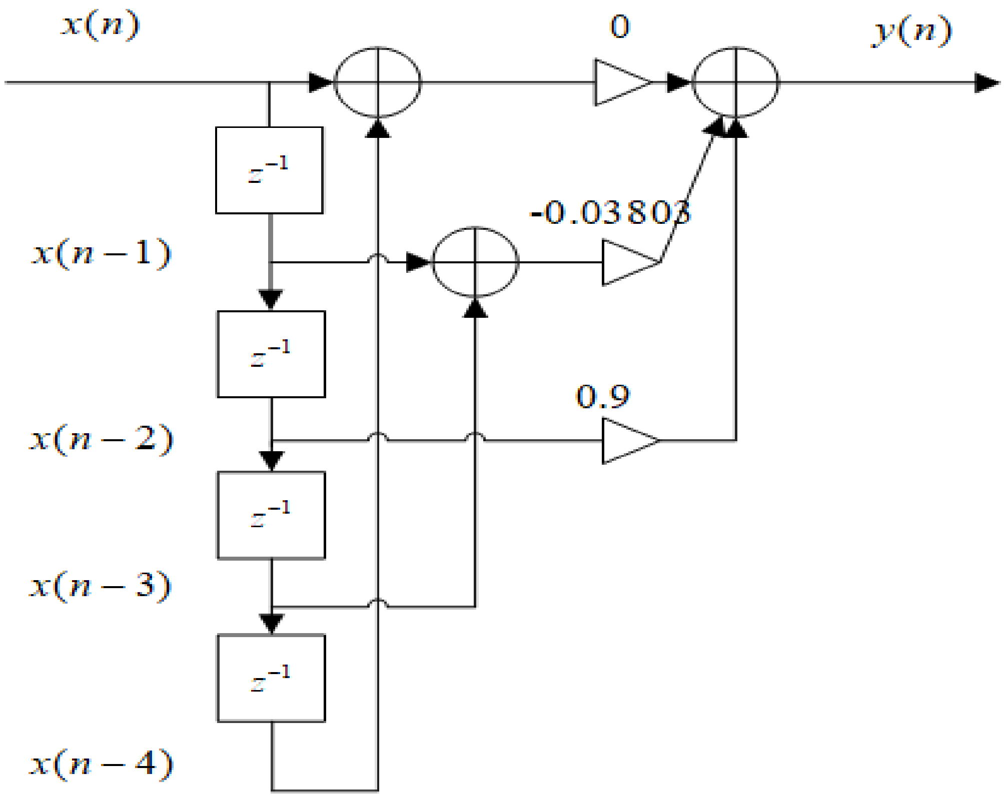


e. Draw the realization block diagram for linear phase implementation



Problem 3

Given a sampling rate of 2000 Hz, design a 3-tap FIR low-pass filter with a cut-off frequency of 100 Hz using the frequency sampling method. Determine the transfer function H(z) and difference Equation.

(15 points)

solution

$M = \frac{N-1}{2} = 1, 0 < \frac{f_c}{f_s/(2M)} = 0.1 < 1$

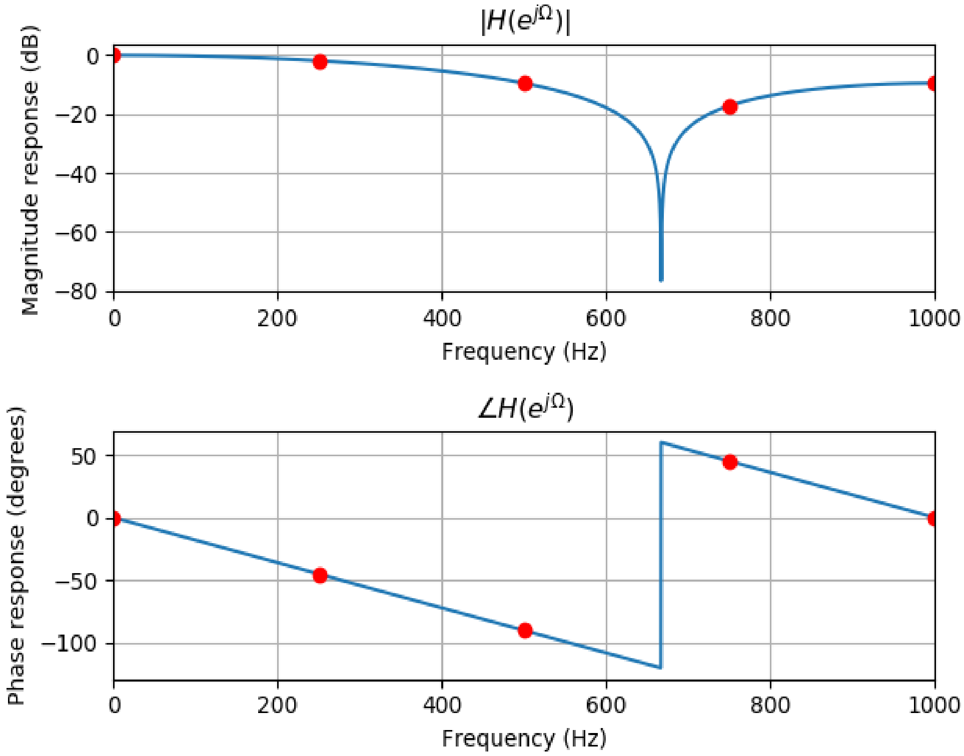
So, $H_k = [1, 0] \quad \text{for } k = 0, \ldots, M$

Then, calculate $h(-M), \ldots, h(0), \ldots, h(M)$

$$b_{n+M} = h(n) = \frac{1}{2M+1}\left\{H_0 + 2\sum_{k=1}^{M} H_k \cos\left(\frac{2\pi kn}{2M+1}\right)\right\} \quad \text{for } n = -M, \cdots, 0, \cdots, M$$

Thus, $h(-M) \sim h(M) = [0.3333, 0.3333, 0.3333]$

The transfer function $H(z) = 0.3333 + 0.3333z^{-1} + 0.3333z^{-2}$



Problem 4

Design a first-order high-pass IIR digital Butterworth filter with a cut-off frequency of 400 Hz at a sampling frequency of 1000 Hz using the bilinear transformation method. (25 points)

(hint: do not forget the frequency warping)

a. Determine the transfer function H(z).

b. Make a pole-zero plot and determine the stability.

d. Determine difference equations.

e. Set up MATLAB routine “freqz()” to obtain the frequency response plot.

f. Determine the DSP equations for the direct-form II implementation.

g. Draw the realization block diagram using the direct-form II implementation.

solution

a. Determine the transfer function H(z).

$\omega_{zp} = 2\pi \times 400$ rad/s, $f_s = 1000$ Hz, $\omega_{sp} = 2f_s \tan(\frac{\omega_{zp}}{2f_s}) = 6155.3671$

The 1st-order digital lowpass Butterworth filter $H(s') = \frac{1}{s'+1}$

Then substitute $s' = \frac{\omega_{sp}}{s}$, band-pass filter

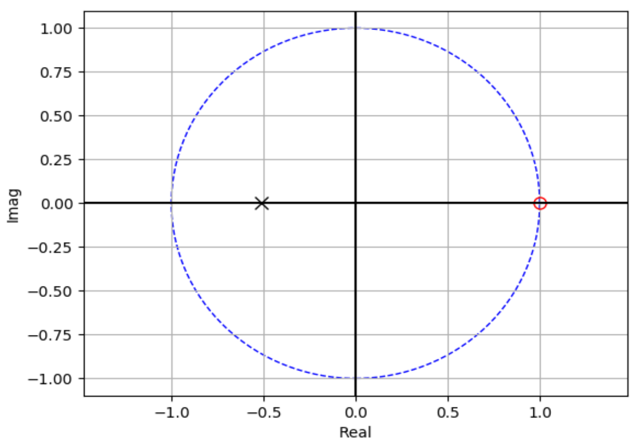
$$H(s) = H(s')|_{s'=\frac{\omega_{sp}}{s}} = \frac{s}{s + 6155.3671}$$

Then with BLT, transfer function

$$H(z) = H(s)|_{z=2f_s\frac{1-z^{-1}}{1+z^{-1}}} = \frac{0.2452 - 0.2452z^{-1}}{1 + 0.5095z^{-1}}$$

b. Make a pole-zero plot and determine the stability.

Pole: $[-0.5095]$ Zero: $[1]$. Because all pole $|-0.5095| < 1$, **Stable**



d. Determine difference equations

$$y(n) = 0.2452x(n) - 0.2452x(n-1) - 0.5095y(n-1)$$

e. Set up MATLAB routine “freqz()” to obtain the frequency response plot.

```
freqz([0.2452, -0.2452], [1, 0.5095], 4096, fs)
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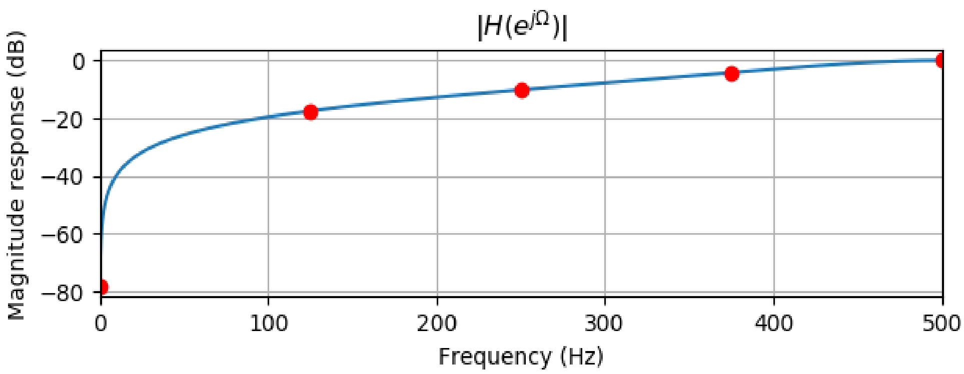
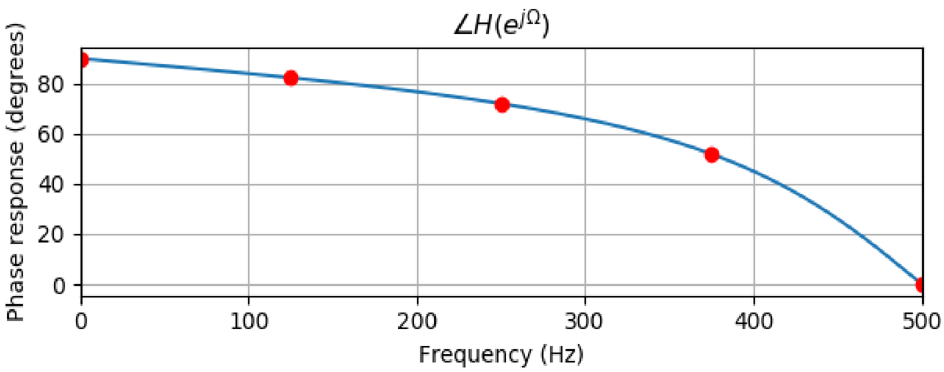
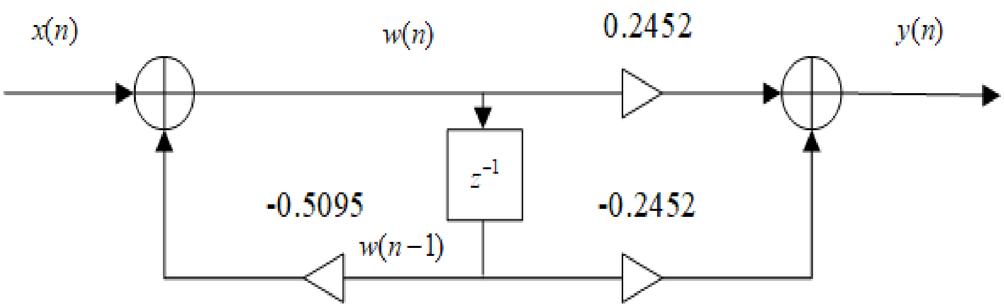




f. Determine the DSP equations for the direct-form II implementation.

$$w(n) = x(n) - 0.5095w(n-1)$$
$$y(n) = 0.2452w(n) - 0.2452w(n-1)$$

g. Draw the realization block diagram using the direct-form II implementation.



Problem 5

Design a second-order band-stop filter (notch filter) using the pole-zero placement method sampling rate=4000 Hz, center frequency=600 Hz, bandwidth=20 Hz

a. Make a pole-zero plot for pole-zero placement.

b. Determine the transfer function H(z)

c. Draw the realization black diagram in the direct form I.

(15 points)

solution

a. Make a pole-zero plot for pole-zero placement.

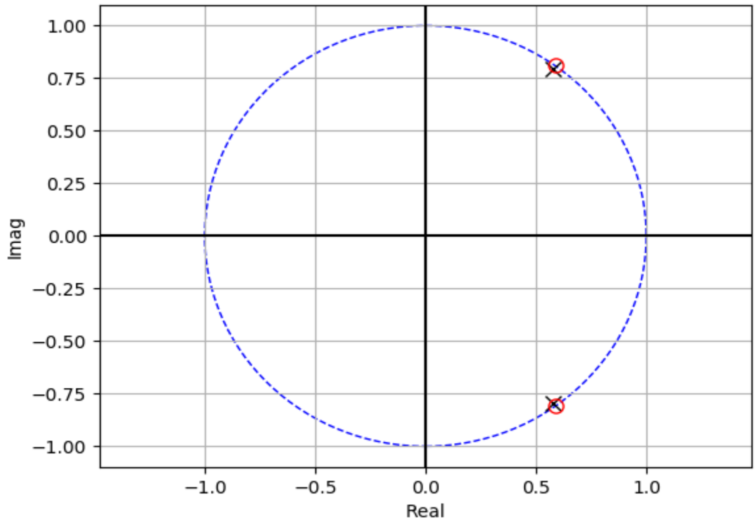
$$\begin{aligned} H(z) &= K\frac{(z-e^{j\theta_0})(z+e^{-j\theta_0})}{(z-r_0e^{j\theta_0})(z-r_0e^{-j\theta_0})} \\ &= K\frac{z^2-2\cos(\theta_0)z+1}{z^2-2r_0\cos(\theta_0)z+r_0^2} \end{aligned}$$

First, compute r_0, θ_0

$$\begin{aligned} r_0 &= 1-2\pi\times(0.5\frac{BW}{f_s}) = 0.9843 \\ \theta_0 &= 2\pi\times(\frac{f_{\text{center}}}{f_s}) = 0.9425 = 54^\circ \end{aligned}$$

Pole: $[r_0e^{j\theta_0}, r_0e^{-j\theta_0}] = [0.5786 - j0.7963, 0.5786 + j0.7963]$

Zero: $[e^{j\theta_0}, e^{-j\theta_0}] = [0.5878 - j0.8090, 0.5878 + j0.8090]$



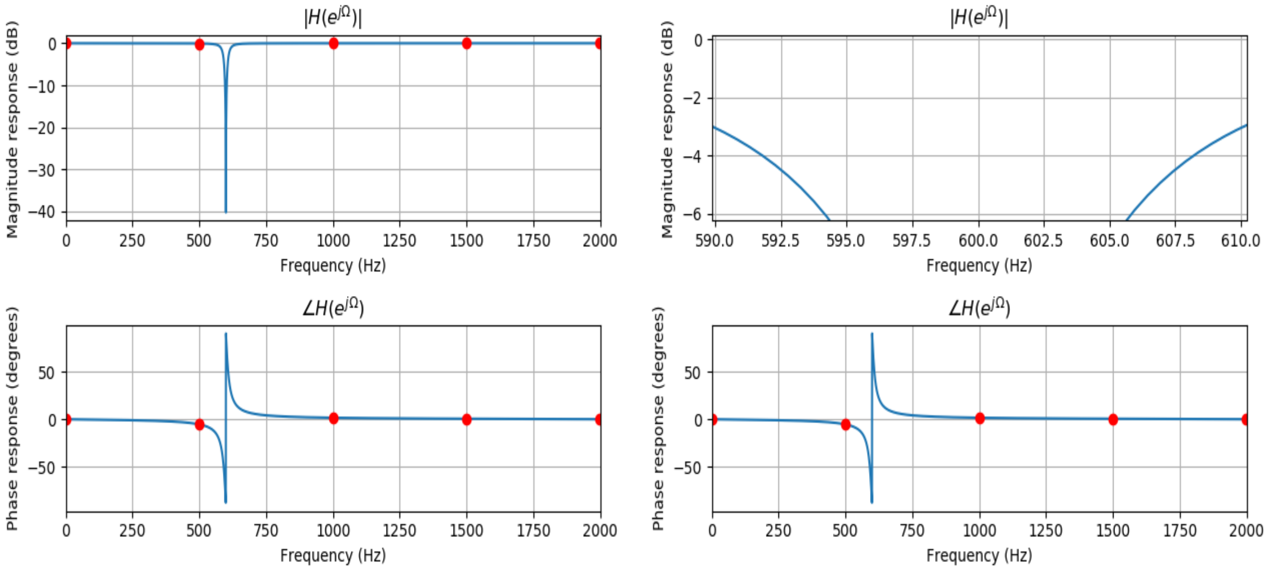
b. Determine the transfer function H(z)

Then compute K

$$K = \frac{(1 + r_0^2) - 2r_0 \cos(\theta_0)}{2 - 2\cos(\theta_0)} = 0.9846$$

Thus, transfer function

$$H(z) = \frac{0.9846 - 1.1575z^{-1} + 0.9846z^{-2}}{1 - 1.1571z^{-1} + 0.9688z^{-2}}$$



c. Draw the realization black diagram in the direct form I.

$$y(n) = 0.9846x(n) - 1.1575x(n-1) + 0.9846x(n-2) + 1.1571y(n-1) - 0.9688y(n-2)$$