Steady Error

assumption: it has been verified to be stable by the method of Routh-Hurwitz Criterion

if D = 0

$$\begin{aligned} e_{ss} &= \lim_{t\to+\infty}[r(t)-c(t)] \\ &= \lim_{s\to0+} s[R(s)-C(s)] \\ &= \lim_{s\to0+} sR(s)[1-T(s)] \\ &= \lim_{s\to0+} sR(s)[1-C(sI-A)^{-1}B] \end{aligned}$$

if $r(t)=u(t)$, step function, $R(s)=\frac{1}{s}$

$$\begin{aligned} e_{ss} &= \lim_{s\to0+}[1-C(sI-A)^{-1}B] \\ &= [1-C(0\cdot I-A)^{-1}B] \\ &= [1+CA^{-1}B] \end{aligned}$$

if $r(t)=tu(t)$, here $R(s)=\frac{1}{s^2}$ only when $[1+CA^{-1}B]=0$, the Steady Error e_{ss} exists

$$\begin{aligned} e_{ss} &= \lim_{s\to0+}\frac{[1-C(sI-A)^{-1}B]}{s} \\ &= \lim_{s\to0+}\frac{[1-C(-A)^{-1}B]}{s} - \frac{[C(sI-A)^{-1}B-C(-A)^{-1}B]}{s} \\ &= \lim_{s\to0+}\frac{[1+CA^{-1}B]}{s} - C\lim_{s\to0+}\frac{[(sI-A)^{-1}-(-A)^{-1}]}{s}B \\ &= \lim_{s\to0+}\frac{[1+CA^{-1}B]}{s} - C\lim_{s\to0+}\frac{d(sI-A)^{-1}}{ds}B \\ &= \lim_{s\to0+}\frac{[1+CA^{-1}B]}{s} + C\lim_{s\to0+}(sI-A)^{-2}B \\ &= \lim_{s\to0+}\frac{[1+CA^{-1}B]}{s} + CA^{-2}B \end{aligned}$$