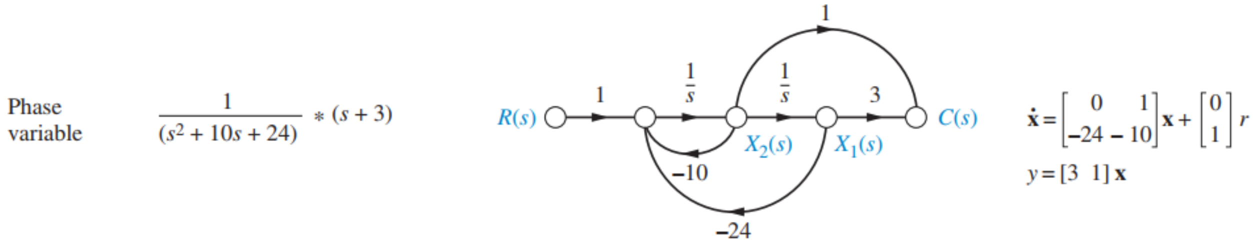
5.6 Signal-Flow Graphs of State Equations

P254 of edition 7 book
Forms:

1. Phase variable
2. Cascade
3. Parallel
4. Controller canonical
5. Observer canonical

$$G(s) = \frac{b_m s^m + \ldots + b_0}{s^n + a_{n-1} s^{n-1} + \ldots + a_0}$$

Phase variable



define $X_1(s) \equiv R(s)/(s^n + a_{n-1}s^{n-1} + \ldots + a_0)$ \
Then define $X_2(s) = sX_1(s), \cdots, X_n(s) = sX_{n-1}(s)$\
So $X_k(s) = s^{k-1}X_1(s)$

then

$$R(s) = (s^n + a_{n-1}s^{n-1} + \ldots + a_0)X_1(s)$$
$$= sX_n(s) + (a_{n-1}X_n(s) + \ldots + a_0X_1(s))$$

rearrange it

$$sX_k(s) = X_{k+1}(s)$$
$$sX_n(s) = -(a_0X_1(s) + \ldots + a_{n-1}X_n(s)) + R(s)$$

Moreover

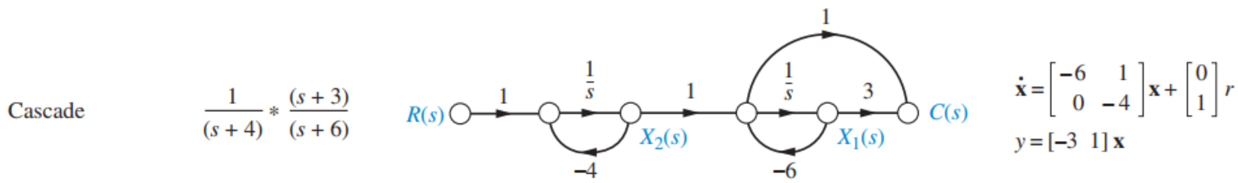
$$C(s) = (b_m s^m + \ldots + b_0)X_1(s)$$
$$= b_m X_{m+1}(s) + \ldots + b_0 X_1(s)$$

So

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -a_0 & -a_1 & -a_2 & \cdots & -a_{n-1} \end{bmatrix} B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$
$$C = [b_0 \quad \cdots \quad b_{m+1} \quad \cdots \quad 0]$$

and $\dot{X} = AX + Br, Y = CX$

Cascade



$$G(s) = \prod \frac{1}{s - s_k} \prod \frac{s - p_k}{s - s_k} * K$$

define $X_n(s) = R(s)\frac{1}{s - s_n}$, here $sX_n(s) = s_n X_n(s) + R(s)$

define $A_k(s) = A_{k+1}(s)\frac{s-p_k}{s-s_k}$

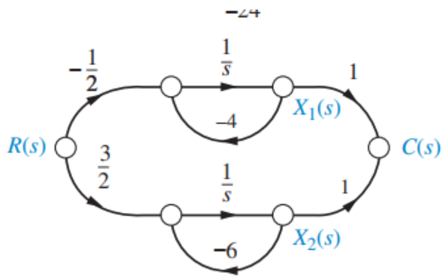
moreover

$$A_{k+1}(s) + s_k X_k(s) = sX_k(s)$$
$$sX_k(s) - p_k X_k(s) = A_k(s)$$

so

$$sX_k(s) = p_k X_k(s) + A_k(s)$$
$$= p_k X_k(s) + (s - s_{k-1})X_{k-1}(s)$$
$$s[X_k(s) - X_{k-1}(s)] = p_k X_k(s) - s_{k-1} X_{k-1}(s)$$

Parallel

Parallel	$\frac{-1/2}{(s+4)} + \frac{3/2}{s+6}$		$\dot{\mathbf{x}} = \begin{bmatrix} -4 & 0 \\ 0 & -6 \end{bmatrix}\mathbf{x} + \begin{bmatrix} -\frac{1}{2} \\ \frac{3}{2} \end{bmatrix} r$ $y = [1 \;\; 1]\,\mathbf{x}$

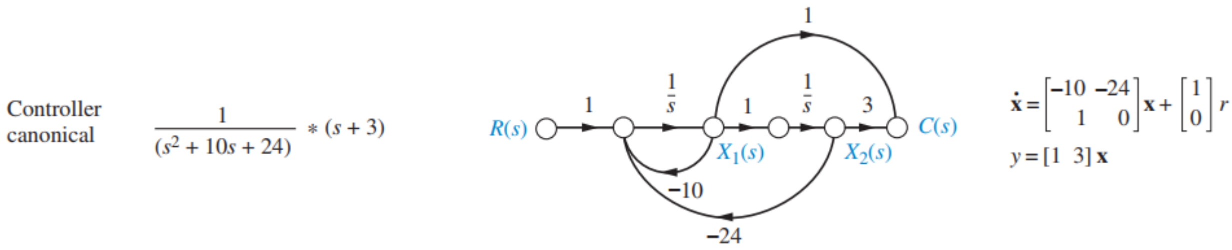
$$G(s) = \sum \frac{t_k}{s - s_k}$$

then define $X_k(s) \equiv R(s)\frac{t_k}{s-s_k}$

so

$$sX_k(s) = s_k X_k(s) + t_k R(s)$$
$$Y(s) = \sum X_k(s)$$

Controller canonical



reverse $X_1, \ldots, X_n$ to $X_n, \ldots, X_1\backslash$
So the matrix A is flipped across the main diagonal

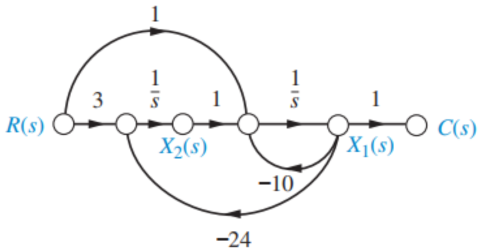
$$A = \begin{bmatrix} -a_{n-1} & \cdots & -a_2 & -a_1 & -a_0 \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix} B = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}$$
$$C = [0 \quad 0 \quad \cdots \quad b_{m+1} \quad \cdots \quad b_0]$$

and $\dot{X} = AX + Br, Y = CX$

Observer canonical

Observer canonical

$$\frac{\frac{1}{s}+\frac{3}{s^2}}{1+\frac{10}{s}+\frac{24}{s^2}}$$



$$\dot{\mathbf{x}}=\begin{bmatrix}-10 & 1\\ -24 & 0\end{bmatrix}\mathbf{x}+\begin{bmatrix}1\\ 3\end{bmatrix}r$$
$$y=[1\quad 0]\,\mathbf{x}$$

reverse $C(s) \Leftrightarrow R(s)$\
reverse $X \Leftrightarrow \dot{X}$\
change direction of arrows

$$G(s)=C(sI-A)^{-1}B$$
$$=G(s)^T=B^T(sI-A^T)^{-1}C^T$$

So here $A \Leftarrow A^T$\
$B \Leftarrow C^T$\
$C \Leftarrow B^T$\
So

$$A=\begin{bmatrix}-a_{n-1} & 1 & 0 & 0 & 0\\ -a_{n-2} & 0 & 1 & 0 & 0\\ \vdots & \vdots & \vdots & \vdots & \vdots\\ -a_1 & 0 & \cdots & 0 & 1\\ -a_0 & 0 & \cdots & 0 & 0\end{bmatrix} B=\begin{bmatrix}0\\ 0\\ \vdots\\ b_{m+1}\\ \vdots\\ b_0\end{bmatrix}$$
$$C=[1\quad 0\quad \cdots\quad 0\quad 0]$$

and $\dot{X}=AX+Br, Y=CX$