Lecture 2

Lecture 2- State Space Representation

symbols

$\dot{X}(t) = AX(t) + BU(t)$
$Y(t) = CX(t) + DU(t)$

$\dot{X}(t)$ derivative of state vector

$X(t)$ state vector | nx1

$Y(t)$ output vector | px1

$U(t)$ input/control vector | mx1

A system matrix | nxn

B input matrix | nxm

C output matrix | pxn

D feedforward matrix | pxm

set $X(t)|_{t=0} = 0$, do Laplace Transform
$sX(s) = AX(s) + BU(s)$
$Y(s) = CX(s) + DU(s)$

so,
$X(s) = (sI - A)^{-1}BU(s) = \frac{adj(sI-A)B}{det(sI-A)}U(s)$
$Y(s) = [\frac{Cadj(sI-A)B}{det(sI-A)} + D]U(s)$

thus, transfer function $G(s)$
$G(s) \equiv \frac{Y(s)}{U(s)} = \frac{Cadj(sI-A)B}{det(sI-A)} + D$

if $X(t)|_{t=0} = X(0)$

$$\begin{aligned} sX(s) - X(0) &= AX(s) + BU(s) \\ Y(s) &= CX(s) + DU(s) \end{aligned}$$

then

$$Y(s) = C(sI - A)^{-1}X(0) + C(sI - A)^{-1}BU(s) + DU(s)$$

Here set $\Phi(t) \equiv L[(sI - A)^{-1}] = e^{At} = \sum_{k=0}^{\infty} \frac{t^k A^k}{k!}$

In another way

$$(sI - A)^{-1} = \begin{cases} -A^{-1}(I - sA^{-1})^{-1} = -A^{-1}[\sum_{k=0}^{\infty} s^k A^{-k}] & |s| < \lambda_{min} \\ s^{-1}(I - \frac{A}{s})^{-1} = \frac{1}{s}[\sum_{k=0}^{\infty} s^{-k} A^k] & |s| > \lambda_{max} \end{cases}$$

Here $\frac{1}{s^{k+1}} = L^{-1}[\frac{t^k}{k!}]$

So we can obtain $(sI - A)^{-1}|_{s=0} = -A^{-1}, \lim_{s\to\infty} s(sI - A)^{-1} = I$