Chap 2: Linearization

$$y - y_0 = y_0'(x - x_0)$$

example 1

consider such a nonlinear model: $i_r = e^{V_r}$

$$C\frac{dV}{dt} + i_r - i(t) - 2 = 0$$

consider a small Perturbation on the Equilibrium

$$V = V_0 + \delta V$$
$$i_r = e^{V_r} = e^{V_0 + \delta V}$$

Let $C = 1$, we have

$$\frac{d\delta V}{dt} + e^{V_0 + \delta V} - 2 = i(t)$$

substitute $e^{V_0+\delta V} \approx e^{V_0} + e^{V_0}\delta V$, then

$$\frac{d\delta V}{dt} + e^{V_0} + e^{V_0}\delta V - 2 = i(t)$$

Thus means

$$\frac{d\delta V}{dt} + e^{V_0}\delta V = i(t) + 2 - e^{V_0}$$

Because $i_r|_{V_r=V_0} = e^{V_0} = 2, V_0 = \ln(2) = 0.693$, we obtain

$$\frac{d\delta V}{dt} + 2\delta V = i(t)$$

Laplace transform $\mathbb{L}$

$$(s+2)\delta V(s) - \delta V(0_-) = I(s)$$

that is because of the property of Laplace transform

$$\int_{0_-}^{\infty} \frac{df(t)}{dt} e^{-st} dt = e^{-st} f(t)|_{0_-}^{\infty} + s\int_{0_-}^{\infty} f(t)e^{-st}dt$$

$$\mathbb{L}[\frac{df(t)}{dt}] = s\mathbb{L}[f(t)] - f(0_-)$$

example 2

$$M\frac{d^2y(t)}{dt^2} = Mg - \frac{i^2(t)}{y(t)}$$

$$V(t) = Ri(t) + L\frac{di(t)}{dt}$$

$V(t)$ is input, now choose the names of variables as

$$X_1 = y(t)$$
$$X_2 = \frac{dy(t)}{dt}$$
$$X_3 = i(t)$$

thus, we obtain

$$\dot{X}_1 = X_2$$
$$\dot{X}_2 = -\frac{X_3^2}{MX_1} + g$$
$$\dot{X}_3 = -\frac{R}{L}X_3 + L^{-1}V(t)$$

suppose input $V(t) = V_0 + \Delta V(t), i(t) = i_0 + \Delta i(t), y(t) = y_0 + \Delta y(t)$, then let $\dot{X} = 0$

$$0 = \frac{dy(t)}{dt}|_{y(t)=y_0}$$
$$0 = Mg - \frac{i_0^2}{y_0}$$
$$0 = -Ri_0 + V_0$$

we can write down the equilibrium for input V_0

$$i_0 = \frac{V_0}{R}$$
$$y_0 = \frac{i_0^2}{Mg} = \frac{V_0^2}{R^2Mg}$$

It implies that

$$
\begin{aligned}
X_{10} &= y_0 = \frac{V_0^2}{R^2 Mg} \\
X_{20} &= \frac{dy(t)}{dt}|_{y(t)=y_0} = 0 \\
X_{30} &= i_0 = \frac{V_0}{R}
\end{aligned}
$$

now define $\delta X \equiv X - X_0, \delta V = V - V_0$

$$
\begin{aligned}
\dot{\delta} X_1 &= \delta X_2 \\
\dot{\delta} X_2 &= \frac{\partial[-\frac{X_3^2}{MX_1}]}{\partial X_1}|_{X=X_0}\delta X_1 + \frac{\partial[-\frac{X_3^2}{MX_1}]}{\partial X_3}|_{X=X_0}\delta X_3 \\
&= [\frac{X_3^2}{MX_1^2}]|_{X=X_0}\delta X_1 + [-\frac{2X_3}{MX_1}]|_{X=X_0}\delta X_3 \\
&= \frac{R^2 Mg^2}{V_0^2}\delta X_1 + [\frac{-2Rg}{V_0}]\delta X_3 \\
\dot{\delta} X_3 &= -\frac{R}{L}\delta X_3 + \delta V(t)
\end{aligned}
$$

therefore

$$
\dot{\delta} X = \begin{bmatrix} 0 & 1 & 0 \\ \frac{R^2 Mg^2}{V_0^2} & 0 & \frac{-2Rg}{V_0} \\ 0 & 0 & -\frac{R}{L} \end{bmatrix} \delta X + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \delta V
$$

Then calculate the poles of linearized model: if poles are all in left plane <=> the model is stable!!!

Is it Controllable or Not?

$$
|C_M| = [B \quad AB \quad A^2 B] \; = \begin{bmatrix} 0 & 0 & \frac{-2Rg}{V_0} \\ 0 & \frac{-2Rg}{V0} & \frac{2R^2 g}{LV0} \\ 1 & -\frac{R}{L} & \frac{R^2}{L^2} \end{bmatrix} \neq 0
$$

If so, we could control it to ensure the model stable